Modelling and self-tuning pressure control in an injection molding cavity

Alfredo E. Varela Villalobos

Facultad de Ingeniería, Escuela de Ingeniería Química, Universidad de Carabobo, Apartado 129. Valencia 2001, Venezuela.
Tel/Fax: 41-674828. E-mail: avarela@thor.uc.edu.ve

Abstract

Injection molding of thermoplastics is a cyclical process where a granular polymer is melted and injected into the cavity, molded under pressure and ejected after solidification. The control of cavity pressure profile from cycle to cycle during the filling stage is important to ensure quality injection molded parts. First, open-loop experiments are performed to determine and verify an appropriate model order. Secondly, due to the time-varying characteristics of this process, a self-tuning algorithm with an observer is employed for controlling the cavity pressure time profile to a set-point trajectory. The model parameters are determined on-line using the recursive least-squares estimation algorithm, and the controller parameters are calculated using the pole location procedure. In order to reduce the controller saturation, the self-tuning algorithm is implemented along with a first-order observer and state feedback.

Key words: Adaptive control, cavity pressure control, self-tuning control, injection molding.

Modelaje y control auto-ajustable de la presión en una cavidad de moldeo por inyección

Resumen

El moldeo de termoplásticos por inyección es un proceso cíclico en el que un polímero granular es fundido y luego inyectado dentro de una cavidad, moldeado a presión y finalmente eyectado después de su solidificación. El control del perfil de presión en la cavidad es importante para el control de calidad del producto moldeado. Primero se realizaron experimentos a lazo abierto con el fin de determinar y seleccionar un orden apropiado del modelo. Posteriormente, debido a la naturaleza variante de este proceso, se implementó una estrategia de control auto-ajustable ("self-tuning") para controlar el perfil de presión a una trayectoria de referencia. Los parámetros del modelo son determinados en línea mediante un algoritmo de mínimos cuadrados, y los parámetros del controlador son calculados usando el procedimiento de ubicación de polos. Para reducir la saturación del controlador, el algoritmo auto-ajustable es implementado en conjunto con un observador de primer orden y un realimentador de estados.

Palabras clave: Control adaptivo, control self-tuning, moldeo por inyección.

Introduction

The cavity pressure is the primary factor affecting the final part quality of an injection-molding machine. Cycle-to-cycle variations of the mold-cavity temperature and pressure during the molding process may produce warpage, due to residual stresses, and changes in physical properties of the parts with tight tolerance limits.

Several researchers have pointed out the importance of measuring and controlling the cavity pressure. The paper of Kamal *et al.* [1] is worth mentioning because it contains early studies of the cavity pressure dynamics and control. Costin *et al.* [2] applied the self tuning regulator algorithm (STR) to control the hydraulic pressure to a constant pressure gradient in the filling stage.

Gao [3] implemented the STR in the control of the cavity pressure in the filling and packing. Adaptive control was also used by Chiu *et al.* [4]. Smud *et al.* [5] used several algorithms to control the cavity pressure during packing and holding phases using the clamp force as manipulated variable.

Cycle-to-cycle variations of the mould-cavity temperature and pressure during the moulding process may produce warpage, due to residual stresses, and changes in physical properties of the parts with tight tolerance limits.

The plant Figure 1 was described elsewhere [6] A 3-mm thick rectangular cavity with length 10.1 cm and width 6.5 cm was used in this study. A pressure sensor is installed flush with the cavity surface of the fixed plate at the gate. The control variable u is the supply servo-valve (SSV) opening, and p_h, p_n, and p refer to the hydraulic, nozzle and cavity gate pressure, respectively.

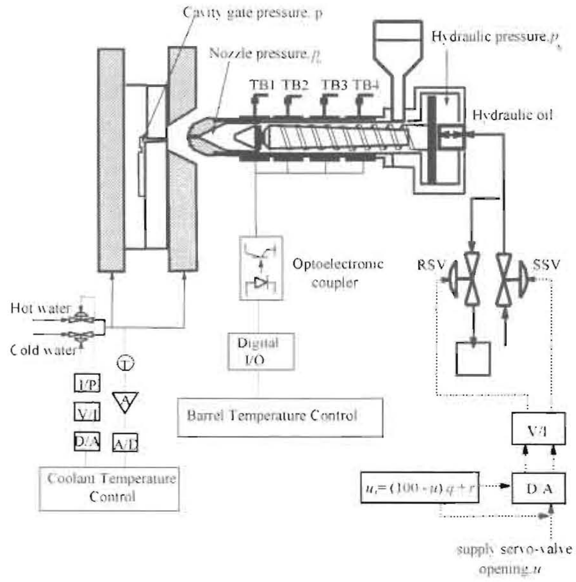

Figure 1. Schematic of the injection molding system.

Process Model

Preliminary experiments were conducted to record input/response data that could be used in selecting the process model structure. Preliminary experimental results suggested that a first-order model could describe the response in hydraulic pressure for a fixed servo-valve opening (u)

$$\tau_h \frac{dP_h}{dt} + P_h = K_h u \tag{1}$$

The following assumptions are used to derive simple models: 1) the frictional force opposing screw movement is negligible; 2) the acceleration of the actuator-screw assembly is proportional to the hydraulic pressure gradient, dp_h/dt; 3) the polymer does not leak back through the injection valve; and 4) the polymer flow in the nozzle and runner is isothermal. Assumptions 1) and 2) imply that a force balance for the screw-assembly may be written as

$$A_h p_h - p_h A_b = K \frac{dp_h}{dt} \tag{2}$$

where A_h and A_n are the effective areas for the hydraulic and nozzle pressure. A change in polymer mass in the cavity equals the mass flowing through the runner, so a mass balance for the cavity may be expressed as

$$V_c \frac{\partial \rho}{\partial t} = \rho_n \left(\frac{p_n - p}{R_r} \right) \tag{3}$$

where R_r is the flow resistance from the nozzle to the cavity gate. The variations in density with time is expressed as a function of the cavity gate pressure and temperature variations as

$$\frac{\partial \rho}{\partial t} = \left(\frac{\partial \rho}{\partial p} \right)_T \left(\frac{\partial p}{\partial t} \right) + \left(\frac{\partial \rho}{\partial T} \right)_P \left(\frac{\partial T}{\partial t} \right) \tag{4}$$

Assuming isothermal filling, supposition (4), and substituting the density variations from equation (4) into equation (3) gives

$$\frac{1}{c} \frac{dp}{dt} = p_n - p \tag{5}$$

where

$$c = \left[\left(\frac{\partial \rho}{\partial p} \right)_T \right]^{-1} \frac{\rho_n}{V_c R_r} \tag{6}$$

Considering constant τ_h, K_h, A_n, K, and c, which is valid for short periods only, and solving equations (1), (2), and (5) by Laplace transformation, the following expression results for the cavity pressure dynamics

$$\frac{P(s)}{U(s)} = \frac{K_p(\tau_a s + 1)}{(1+\tau_1 s)(1+\tau_2 s)} \quad (7)$$

where $\tau_1 = \tau_h$, $\tau_2 = 1/c$, $K_p = A_h/A_n$, and $\tau_a = mK/A_h$. The discrete transfer function with zero-order hold is

$$\frac{y(k)}{u(k)} = \frac{b_o z^{-1} + b_1 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (8)$$

and the corresponding difference equation for a constant input is

$$y(k) + a_1 y(k) + a_2 y(k-2) = b_o u(k-1) + b_1 u(k-2) \quad (9)$$

Equation (9) shows a second-order model for the cavity pressure response. However, preliminary experimental results suggest that the process can be approximated to an overdamped second-order model, which gives similar results to a first-order model. The discrete transfer functions with zero order hold for a first order model is

$$\frac{y(k)}{u(k)} = \frac{b_1 z^{-1}}{1 + a_1 z^{-1}} \quad (10)$$

where

$$a_1 = -e^{-\Delta t/\tau}, \quad b_1 = (1 + a_1)K/\tau \quad (11)$$

In discrete-time domain, equation (10) is expressed as

$$y(k) + a_1 y(k-1) = b_1 u(k-1) \quad (12)$$

Parameters in equations (9) and (12) should be estimated on-line using input/output process data, as they vary with time. For low-order systems with time-varying parameters, an appropriate estimation technique is the least-squares algorithm with an exponential forgetting factor. This identification algorithm has been given by Åström and Wittenmark [7] is described below.

Assume a process is described by the following linear difference equation with constant parameters:

$$A(z^{-1})y(k) = B(z^{-1})u(k-1) + e(k) \quad (13)$$

polynomials A and B are given by

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_{na} z^{-na}$$

$$B(z^{-1}) = b_1 + b_2 z^{-1} + \ldots + a_{nb} z^{-nb} \quad (14)$$

where z is the unit forward shift operator. To find coefficients of the polynomials in equation (6), equation (5) is conveniently written in the matrix form

$$y(k) = \varphi^T(k)\theta + e(k) \quad (15)$$

where φ^T is the vector of measured values of output and input variables

$$\varphi^T(k) = [-y(k-1) \ldots y(k-n_a) \quad u(k-1) \ldots u(k-n_b-1)] \quad (16)$$

and θ is the parameter vector

$$\theta^T = [a_1 \ldots a_{na} \; b_1 \ldots b_{nb}] \quad (17)$$

The parameters are calculated by finding the minimum of the function $J(\theta, k)$, defined as

$$J(\theta, k) = \frac{1}{2}\sum_{i=1}^{k} \lambda^{k-i}\left[y(k) - \varphi^T(k)\theta\right]^2 \quad (18)$$

where λ is the forgetting factor. The least-squares solution [13] is obtained with:

$$\theta(k) = \theta(k-1) + K(k)\left[y(k) - \varphi^T(k)\theta(k-1)\right]$$

$$K(k) = P(k-1)\varphi(k)\left(\lambda + \varphi^T(k)P(k-1)\varphi(k)\right)^{-1}$$

$$P(k) = \left[I - K(k)\varphi^T(k)\right]P(k-1)/\lambda \quad (19)$$

Values between 0 and 1 are given to the forgetting factor(λ). In injection molding, λ is appropriately selected to reflect the evolving cavity pressure dynamics.

Self-Tuning Pressure Control

The self-tuning control strategy is shown in Figure 2a, where the servo-valve opening is the manipulated variable. Parameter estimation and

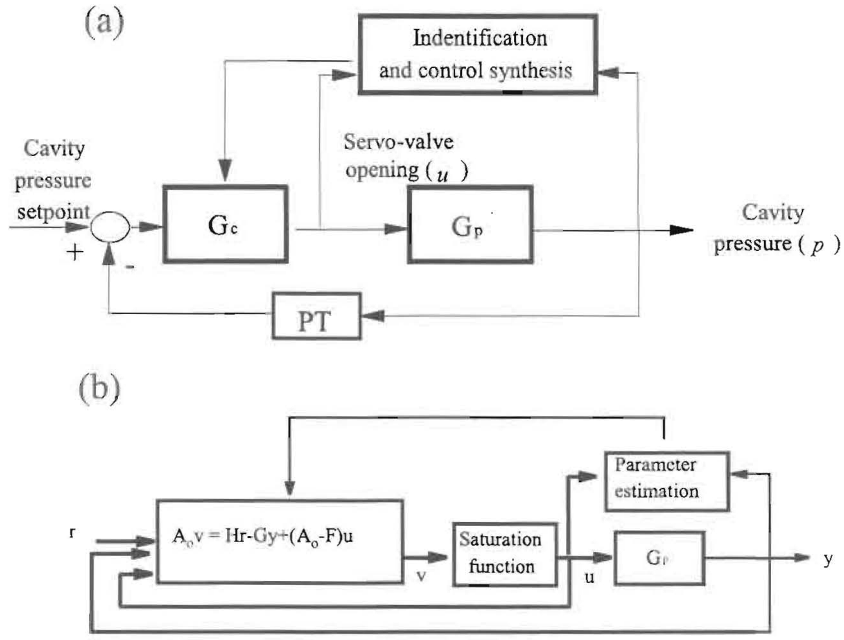

Figure 2. Block diagrams of: (a) The self-tuning cavity pressure control. PT = pressure transducer, Gc = controller transfer function, Gp = process transfer function.
(b) The self-tuning control strategy using an observer.

control syntehis are performed on-line using the algorithms presented the text of Wellstead and Zarrop [8]. These are described briefly as follows.

Neglecting model errors, $e(k)$, in equation (15), this can be written in polynomial form as:

$$A(z^{-1})y(k) = B(z^{-1})z^{-1}u(k) \qquad (20)$$

and the feedback controller is of the form

$$Fu(k) = Hr(k) - Gy(k) \qquad (21)$$

where $r(k)$ and $y(k)$ denote the reference and the measured controlled variable, respectively. F, H, and G are polynomials which are selected so that the system output tracks the reference signal $r(k)$. Substituting $u(k)$ from equation (13) into equation (12) yields the closed loop form

$$y(k) = \frac{BHz^{-1}}{FA + z^{-1}BG} r(k) = \frac{BHz^{-1}}{T} r(k) \qquad (22)$$

The F and G polynomials are found by solving the polynomial identity

$$FA + z^{-1}BG = T(z^{-1}) \qquad (23)$$

The polynomial H is calculated to achieve the desired output, which is:

$$\frac{y(k)}{r(k)} = \left[\frac{BH}{K}\right]_{z=1} = 1 \qquad (24)$$

For a desired closed-loop response based upon a first-order model, the polynomial $T(z^{-1})$ is written as

$$T(z^{-1}) = 1 - t_1 z^{-1}, \quad t_1 = \exp(-\Delta t / \beta) \qquad (25)$$

where, Δt is the sampling interval, and β the time constant for the desired closed-loop response. Applying the pole location design procedure with the first-order model given by equation (12), the polynomials H and G defined in equation (21) are found as:

$$G = g_o = -\frac{t_1 + a_1}{b_1}, \quad H = h = \frac{1 - t_1}{b_1} \qquad (26)$$

and the controller output is generated as:

$$u(k) = -g_o y(k) + hr(k) \qquad (27)$$

The closed-loop model parameters are estimated using the cavity pressure response, $y(k)$, as output and the servo-valve opening, $u(k)$, as input. Therefore, the controller parameters, g_o and h, are adjusted on-line with the feedback signals from the cavity pressure measurements.

Self-tuning control with observer

Controller saturation is a problem encountered in the control of the cavity pressure. To reduce this problem, employing a controller with observer and state feedback [7] is suggested. The block diagram of the control system with this structure is shown in Figure 2b. The controller equation is obtained from equation (21) in observer form as:

$$A_o(z^{-1})v(k) = H(z^{-1})r(k) - G(z^{-1})y(k) + \left[A_o(z^{-1}) - F(z^{-1})\right]u(k) \qquad (28)$$

where $A_o(z^{-1})$ is the observer polynomial. An observer with first-order dynamic is written as

$$A_o(z^{-1}) = 1 - a_o z^{-1}, \; a_o = \exp(-\Delta t / \tau_o) \qquad (29)$$

where t_o is the observer time constant. Therefore, to reduce controller saturation, a preliminary value $v(k)$ is calculated by:

$$v(k) = -a_o v(k-1) + \frac{1-t_1}{h_1}\left[r(k) + a_o r(k-1)\right] - g_o y(k) + (a_o - h)u(k-1) \qquad (30)$$

and the controller output, $u(k)$, is determined using the saturation function

$$u(k) = \begin{cases} u_{\max}, & \text{if } v(k) > u_{\max} \\ v(k), & \text{if } u_{\min} \le v(k) \le u_{\max} \\ u_{\min}, & \text{if } v(k) < u_{\min} \end{cases} \qquad (31)$$

where u_{max} and u_{min} are the upper and lower bounds of the manipulated variable.

Set-point profile

A linear model for the cavity pressure as a function of time was used to generate a set-point trajectory. This is given by

$$p_{sp}(t) = p_{of} + \frac{p_{ef} - p_{of}}{t_f} t = p_{of} + \left(\frac{dp}{dt}\right)_{sp} t \qquad (32)$$

Table 1
Conditions for the dynamic open-loop experiment

Time settings:	
Injection	13 s
Decompression	2 s
Cooling	10 s
Open	10 s
Coolant temperature set point	40°C
Barrel temperature set points	250/220/200/190°C
Sampling interval	Δ = 0.020 s
Input period (square wave)	T_u = 0.16 s
Input amplitude	A_u = 0.5-70%

where t_f is the filling time, p_{of} the initial pressure, and $(dp/dt)_{sp}$ is the desired slope of the cavity pressure curve during filling. To completely describe the cavity pressure profile, a model for the packing pressure is given by the expression

$$p_p(t) = p_{ef} + (p_{pp} - p_{ef})\left[1 - \exp(-t/\tau_p)\right] \qquad (33)$$

where p_{pp} is the peak pressure and τ_p the time constant for the packing stage.

The slope set-point during filling $(dp/dt)_{sp}$ cannot defined arbitrarily. However, it is possible to find and experimental relationship between the filling slope and the peak pressure which can be used to calculate the set-point for a required peak pressures.

Experimental Procedure

The experiments were divided into three groups: 1) dynamic open-loop experiments, 2) static open-loop experiments, and 3) closed-loop control experiments.

Dynamic open-loop experiment

A square-wave signal with input amplitude of 0.5-70% and period T_u of 0.16 s was employed to obtain data for the model identification. Table 1 summarizes the conditions used to study the dy-

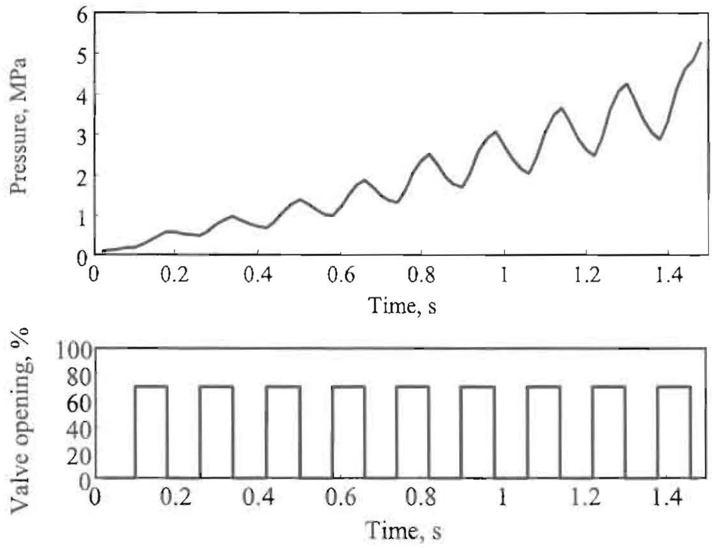

Figure 3. Cavity pressure response to a square-wave variations in servo-valve opening for a dynamic open-loop experiment.

namic of the cavity pressure. The time settings are: injection, interval during which the hydrualic pressure is applied; decompression, period when the supply servo-valve is closed; cooling; and open, the interval for mold opening and plastication. The cavity pressure response during filling for this experiment is presented in Figure 3.

Static open-loop experiment

An open-loop experiment with constant (static) servo-valve opening during each cycle was used to determine a relationship between the slope of the pressure-time curve, during filling, and the peak cavity pressure. Table 2 summarizes the experimental conditions. The experiment was conducted by increasing the servo-valve opening from 10% to 80% every 5 cycles. Figure 4 shows traces of cavity pressures for this experiment. With the increase in the supply servo-valve opening, both the filling slope and the peak pressure increase. From the results, a linear relationship between the peak pressure, p_{pp}, and the filling slope set-points $(dp/dt)_{sp}$, was defined as

$$\left(\frac{dp}{dt}\right)_{sp} = (p_{pp} - 18.95)\frac{1}{0.6} \tag{34}$$

Table 2
Conditions for the static open-loop experiment

Time settings:	
Injection	13 s
Decompression	2 s
Cooling	10 s
Open	10 s
Coolant temperature set point	40°C
Barrel temperature set points	250/220/200/190°C
Data acquisition and servo-valve manipulation	
Sampling interval	$\Delta = 0.040$ s
Servo-valve opening, u (staircase function)	
u (%)	Cycles
10	1 - 5
20	6 -10
40	11 - 15
60	16 - 20

Closed-loop control experiments

The pole location t_1 (see equation (25)) was selected based on the values of the time constant τ_p in equation (33). Values of τ_p were obtained by fitting equation (33) to different data of packing pressure variations with time, and they were found to be between 0.05 s and 0.16 s. Therefore, by considering $\beta = \tau_p$, the desired pole location t_1, for $\Delta t = 0.020$ s, should be between 0.88 and

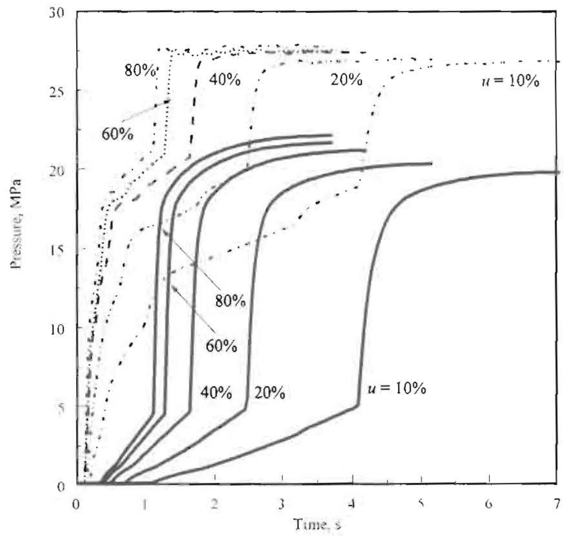

Figure 4. Variations in nozzle and cavity pressures (-) with time using the conditions given in Table 2.

0.67. A value of $t_1 = 0.7$ was chosen for the control experiments. The time constant for the first-order observer was selected as two sampling intervals; $\tau_p = 0.04$ s, which corresponds to $a_o = 0.39$ s (see equation (29)).

The experimental procedure was set using a fixed value of servo-valve opening (open-loop) until the polymer has filled part of the cavity, before starting the self-tuning control action. Experimental conditions for the closed-loop experiments are summarized in Table 3.

Results and Discussion

Data collected in the dynamic open-loop experiment at conditions used in Table 1 was used for the purpose of model identification. The recursive identification algorithm described by Ljung [9] was used to fit the cavity pressure data with the first-order model and second-order models given by equations (12) and (9), respectively.

Two performance criteria [10] were used for model evaluation: the minimum values of the summation of square error (V_N) and the final prediction error (*FPE*), given by:

$$V_N = \frac{1}{N}\sum_{k=1}^{N}\frac{1}{2}\left[y(k) - y_e(k,\theta)\right]^2, \quad FPE = \frac{1+n/N}{1-n/N}V_N \tag{35}$$

where $y_e(k,\theta)$ is the calculated cavity pressure using equations (9) or (12), n is the total number of estimated parameters, and N the length of the data record. The models are compared with re-

Table 3
Conditions for the control of the cavity pressure

Time settings:				
Injection	13 s			
Decompression	2 s			
Cooling	10 s			
Open	10 s			
Coolant temperature set point	40°C			
Barrel temperature set points	240/220/200/190°C			
Sampling interval	$\Delta = 0.020$ s			
Time constants:				
Set-point	$\tau_p = 0.16$			
Observer	$\tau_o = 0.04$			
Other conditions	Experiment			
	CP-1	CP-2	CP-3	CP-4
Forgetting factor, λ	0.95	0.75	0.75	0.75
Input range, u, %	0.5-90	0.5-90	0.5-90	0.5-80
Slope set-points, $(dp/dt)_{sp}$, MPa/s	3.1	3.1	1.74	5.19

Table 4
Comparison between prediction errors for different forgetting factors using conditions of Table 1

First-order model, equation (12)		
λ	V_N (MPa^2)	*FPE* (MPa^2)
0.75	0.147	0.1530
0.90	0.1541	0.1604
0.95	0.1611	0.1677
0.99	0.1683	0.1751
Second-order model, equation(9)		
λ	V_N (MPa^2)	*FPE* (MPa^2)
0.75	2.4539	2.6587
0.90	2.4241	2.6261
0.95	2.3617	2.4002
0.99	2.2996	2.4983

spect to their performance criteria in Table 4, using different forgetting factors. The first-order model showed prediction errors lower than those of the second-order model. These suggest that a first-order model is appropriate. The forgetting factors should be taken between 0.75 and 0.99.

Control of the cavity pressure

Three values of slope set-point profiles were defined for the control of cavity pressure during filling as shown in Table 3. Figure 5 illustrates variations in pressure and servo-valve opening with time for Experiment CP-1 (Table 3). For 10% of initial value of the control signal and λ = 0.95, the cavity pressure tracks the slope set-point (3.10 MPa/s). However, a high value of λ averages parameters in the estimation period and leads to delays in the cavity pressure response with respect to the set-point profile as seen in Figure 5. In addition, parameter variations have been found to be quite rapid during the filling stage. Then, lower forgetting factors are requiered. A value of λ = 0.75 was used for other experiments, as suggested from results in Table 3.

As shown in Figure 7, for Experiment CP-3 with a set-point profile of 1.74 MPa/s, the controller performs better using a lower forgetting factor, although more oscillations occur. This is because low forgetting factors weight the most recent measurements. Figure 8 shows that the response follows the set-point trajectory in Experiment CP-4, and that the control signal remains within the bounds (0.5% < u < 80%) during most of the filling stage. The response is still oscilatory

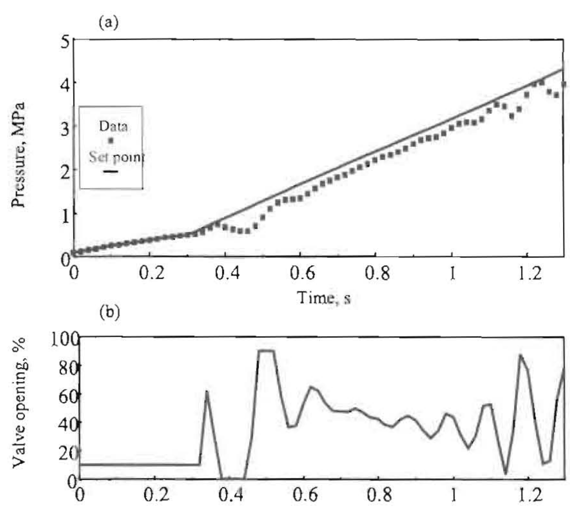

Figure 5. System response during the filling stage in Experiment CP-1 (λ = 0.95), with conditions shown in Table 3.
(a) Cavity pressure and set-point, and
(b) servo-valve opening.

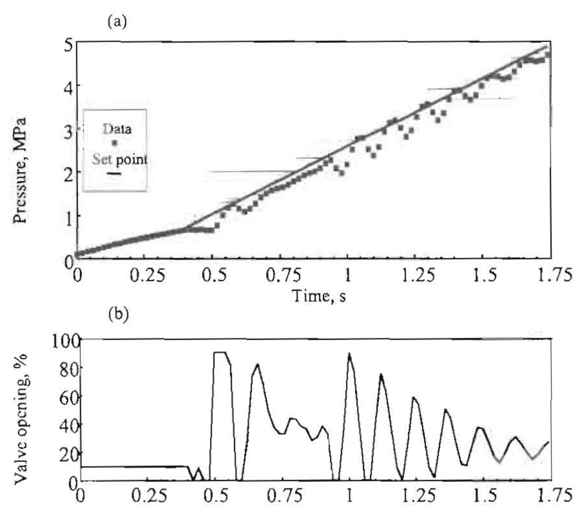

Figure 6. System response during the filling stage in Experiment CP-2 (λ = 0.75), with conditions shown in Table 3.
(a) Cavity pressure and set-point, and
(b) servo-valve opening.

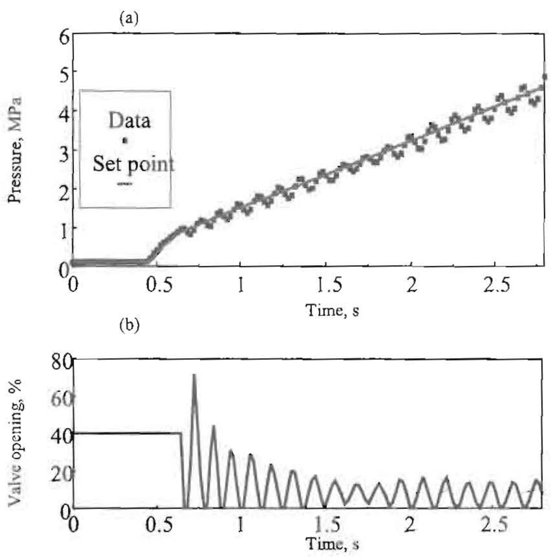

Figure 7. System response during filling and packing in Experiment CP-3 with conditions shown in Table 3. (a) Cavity pressure and set-point, and (b) servo-valve opening.

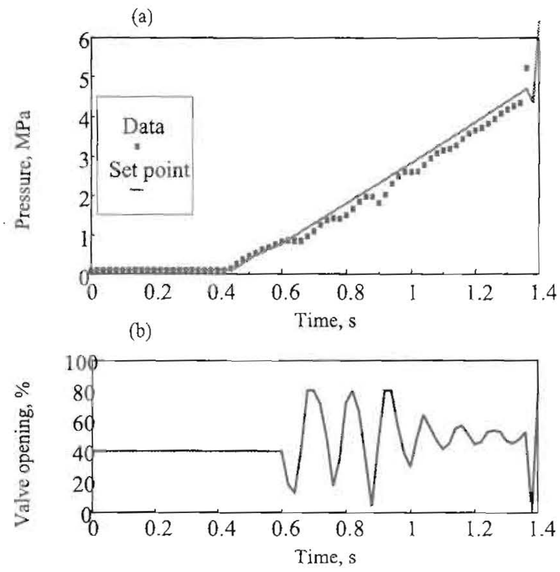

Figure 8. System response during filling stage in Experiment CP-4 with conditions shown inTable 3. (a) Cavity pressure and set-point, and (b) servo-valve opening.

because the upper bound of the servo-valve opening is too high. Oscilations diminished when the upper bound was set at 80% as seen in Figure 8 for Experiment CP-4. Both temperature and pressure change very rapidly during filling, thereby affecting the polystyrene compresibility since it is amorphous and its viscosity is very sensitive to temperature near the glass transition temperature. This leads to oscillations in servo-valve opening. However, reasonable results on the control the cavity pressure profile were obtained.

Conclusions

A first-order model for the cavity pressure dynamics along with a self-tuning approach has been proven to be effective in controlling the cavity pressure profile during the filling stage. Parameters of the model are estimated online using forgetting factor $\lambda = 0.75$, while the controller parameters were determined by the pole location procedure. Controller saturation is reduced using a first-order observer and state feedback. Experimental results showed the effectiveness of the proposed strategy.

The cavity pressure is difficult to control due to polymer solidification in the sprue and runner and at the cavity walls. Polymer solidification causes a damping effect which conducts, in some cases, to oscillations in the manipulated variable and consequently in the cavity pressure. These oscillations may affect part properties. For a future development of this work, to avoid this incovenience, the use of digital filters is suggested.

Acknowledgements

This research was supported by Fundación Gran Mariscal de Ayacucho and Universidad de Carabobo (CDCHT) in Venezuela, as well as by McGill University and the Natural Sciences and Engineering Research Council of Canada (NSERC). Dow Chemical Canada supplied the polystyrene resin used in the experiments.

References

1. Kamal, M.R., Patterson, W.I., Conley, N., Abu Fara, D. and Lohfink, G.: "Dynamic and control of pressure in .the injection molding of thermoplastics". Polym. Eng. Sci., 27 (1987) 1403-1410.
2. Costin, M.H., Okonski, D.A. and Ulicny, J.C.: "Control of an injection molding machine:

Adaptive regulation during filling". Proceedings of the American Control Conference, Piscataway, N.J. USA, (1987) 711-716.

3. Gao, F., Patterson, W.I. and Kamal, M.R.: "Cavity pressure dynamics and self-tunning control for filling and packing phases of thermoplastics injection molding". Polym. Eng. Sci., 36 (1996) 1272 -1285.

4. Chiu, C.P., Wei, J-W and Shih, M-C: "Adaptive model following control of the mold-filling process in an injection molding machine". Polym. Eng. Sci., 31 (1991) 1123-1129.

5. Smud, S.M., Harper, D.O. and Deshpande, P.B: "Advanced process control for injection molding". Polym. Eng. Sci., 31 (1991) 1081-1085.

6. Varela, A.. "Self-tuning pressure control in an injection molding cavity." Proc. of the IASTED Int. Conf., Intelligent System and Control, Oct 28-30, Santa Barbara, California.(1999) 67-72.

7. Åström, K.J. and Wittenmark, B.: "Computer Controlled Systems Theory and Practice". Prentice-Hall, New Jersey, 1990.

8. Wellstead, P.E. and Zarrop, M.B.: "Self -Tuning Systems: Control and Signal Processing". Wiley, New York, 1991.

9. Ljung, L.: "System Identification Toolbox, User's Guide". The Math Works, Inc., Massachusetts, 1991.

10. Ljung, L.: "System Identification, Theory for the User". Prentice-Hall, New Jersey, 1987.

Recibido el 12 de Mayo de 1999
En forma revisada el 26 de Marzo de 2001