Rev. Téc. Ing., Univ. Zulia
Vol. 5 No. 1, 1982.

ESTIMATION OF STRESS-SENSITIVE RESERVOIRS' INITIAL PARAMETERS BY AUTOMATIC HISTORY MATCHING

Rufus O. Elemo
Department of Petroleum Engineering
University of Ibadan
Ibadan, Nigeria.

Roy M. Knapp
School of Petroleum and Geological Engineering
The University of Oklahoma
Norman, Oklahoma.

ABSTRACT

The rock and fluid properties of stress-sensitive reservoirs are dependent on the effective stress on the rock. Previous studies have shown that the currently available conventional well test methods are inadequate for estimating the initial values of stress-sensitive reservoirs' parameters. This paper presents the results of a study made to estimate the initial values of some pertinent parameters for stress-sensitive reservoirs by using the technique of history matching. In automatic history matching, mathematical programming methods are used to optimize a criterion function which is a mathematical formula that characterizes the difference between the calculated model behavior and the observed behavior of the real physical system. The history matching problems in this study were formulated as optimal control problems, that is, the criterion function was minimized by choice or control of the parameters being estimated.

A two dimensional finite difference model of stress-sensitive geopressured-geothermal reservoir flowing a single phase, slightly compressible fluid was developed. The simulator calculated performance data were matched with simulated drawdown and drawdown-buildup data. Single and multi-parameter estimations of the initial formation porosity, permeability and uniaxial compaction coefficient were made. The reliability of the parameter point estimates were determined by constructing the confidence intervals and joint confidence regions of the point estimates and the history and predicted performance data.

The results showed that stress-sensitive parameters can be accurately and reliably estimated by history matching. The single-parameter estimates were most reliable and reliability decreases as the number of unknown parameters increases. Drawdown data are generally preferred to drawdown-buildup data and error-free data are also desirable.

RESUMEN

Las propiedades de rocas y fluídos de yacimientos sensibles a esfuerzo dependen del esfuerzo efectivo sobre la roca. Estudios previos han mostrado que los métodos convencionales corrientemente disponibles de prueba de pozos son inadecuados para la estimación de valores iniciales de parámetros de yacimientos sensibles a esfuerzos. Este trabajo presenta los resultados de un estudio hecho para estimar los valores iniciales de algunos parámetros pertinentes para yacimientos sensibles a esfuerzo, mediante el uso de técnicas de cotejo de historia. En el cotejo automático de historia se utilizan métodos de programación matemática para optimizar una función o criterio, la cual es

una fórmula matemática que caracteriza la diferencia entre el comportamiento calculado del modelo y el comportamiento observado del sistema físico real. En este estudio los problemas de cotejo de historia fueron formulados como problemas del control óptimo, es decir, la función criterio fué minimizada mediante escogencia o control de los parámetros que estaban siendo estimados.

Se desarrolló un modelo bidimensional, de diferencias finitas de yacimiento geotermal y geopresionado, sensible a esfuerzo, con una sola fase fluyendo. Los datos calculados de comportamiento dados por el simulador fueron cotejados con datos de declinación y datos de declinación-restauración. Se hicieron estimaciones de parámetros, simples y múltiples, (dos o más parámetros simultáneamente estimados), de la porosidad inicial de la formación, de la permeabilidad y del coeficiente de compactación uniaxial. La confiabilidad de los estimados puntuales de los parámetros fué determinada estableciendo los intérvalos de confianza y regiones de confianza conjunta de los estimados puntuales y los datos de comportamiento de historia y de predicción.

Los resultados mostraron que los parámetros sensibles a esfuerzo pueden ser precisa y confiablemente estimados por cotejamiento de historia. Los estimados de parámetro simple fueron más confiables y la confiabilidad decrece a medida que el número de parámetros desconocidos aumenta.

En general se prefieren datos de declinación a datos de declinación-restauración y también son deseables datos libres de error.

INTRODUCTION AND PURPOSE

Well test simulations and analyses have demostrated that the currently available well test methods are inapplicable for initial parameter estimation in stress-sensitive reservoirs (Elemo and Knapp, 1979). Methods other than the conventional well test methods have been recommended for any stress - sensitive reservoir parameter estimation. This study was conducted to investigate the estimation of initial reservoir parameters in a stress-sensitive, compacting environment by automatic history matching. History matching is preferred to the methods proposed by Raghavan et al (1972), Samaniego et al (1977) and Evers and Soeiinah (1977) in which a set of transformed differential equation was solved for a dimensionless pseudopressure instead of the classical dimensionless pressure. History matching is preferred because the pseudopressure approach provides only approximate solutions since it is obtained from a linearized form of a nonlinear model. History matching is a parameter estimation technique that considers the model's nonlinearities. The matching was formulated as an optimal control problem, that is, the matching was by choice or control of the parameters in the partial differential equations describing the flow of fluid in the reservoir. A hypothetical compacting geopressured-geothermal aquifer was selected for study. A two-dimensional finite difference model of the aquifer was used. Matches were done with simulated drawdown and drawdown-buildup test performance data with and without normally distributed,independent random measurement errors.

Before a parameter can be effectively estimated by history matching, the temporal response of the performance data to be matched must be sensitive to variations in the parameter. The performance data used for history matching in this study is the reservoir pressure. Elemo and Knapp (1981) have reported that the temporal response of pressure in compacting geopressured-geothermal reservoir is sensitive to variations in the initial formation permeability, uniaxial compaction coefficeint, net pay thickness and the specific productivity index. The temporal response of pressure was reported to be virtually insensitive to the variations in the initial formation porosity. The sensitivities were determined through numerically controlled experiments performed on the parameters of interest. Based on the authors' results, permeability, compaction coefficient and porosity were selected for estimation. The results of various single-parameter estimations of stress-sensitive geopressured-geothermal reservoir permeability, uniaxial compaction coefficient and porosity, whereby only one of the parameters was assumed unknown at a time and to be estimated are reported in this paper. Frequently several reservoir parameters are unknown and must be estimated. This paper also contains the results of multi-parameter estimation runs, whereby more than one of the stress-sensitive reservoir parameters were assumed unknown at a time and to be estimated. Generally, the reservoir was considered to be elastic. However, a few runs were made to examine partially elastic reservoir rock. The elastic case implies that the uniaxial compaction coefficients during loading and unloading are the same while they differ for the partially elastic case. Both homogeneous, isotropic and heterogeneous, anisotropic reservoir systems were considered in this study. Five combinations of multi-parameter estimations were made; these are permeability and compaction coefficient, permeability and porosity, compaction coefficient and porosity, permeability, compaction coefficient and porosity and then partially elastic drawdown and buildup compaction coefficients.

History matching problems are ill-conditioned since there are usually more parameters that are unknown than are the available history data. The problems are therefore statistically underdetermined. Generally, the quantity of data cannot be increased since they are obtained from observation (history). The other possibility is reduction of the number of unknown parameters. The traditional method for parameter reduction is zonation whereby the reservoir is divided into a certain number of zones and a parameter is assumed homogeneous within a zone. Parameterization by the use of sensitivity vectors had been proposed (Elemo(1978)). The reliability of the parameter estimates in this study was checked by constructing confidence limits for the single-parameter point estimates and joint confidence regions for the multi-parameter point estimates. Confidence limits were also established for the historical data and for future performance data generated using the point estimates. It should be noted that the joint confidence regions are better indicators of the reliability of the multi-parameter point estimates because establishment of the regions accounted for the joint variabilities of the parameters. Establishment of the individual confidence limits neglects the cross product(covariances) of the parameters' combinations. The reader is referred to Appendix A for the momentum transport equation, the adjoint system of equations therefrom and other relevant equations developed and used in this study.

The strategy employed in each of the matching runs was to use the optimal control technique to bracket the optimum estimate(s) and then use a direct search method to narrow the range(s) and locate the optimum estimate(s). The mathematical programming method of steepest descent was used in the optimal control segment of the solution. The results showed that geopressured - geothermal reservoir parameters and indeed any stress-sensitive reservoir parameters can be reliably estimated by history matching method. Permeability and compaction coefficient estimates were good but porosity estimates were less reliable. The estimates' reliabilities were indirectly related to the number of parameters being estimated. Heterogeneous systems were found to require many observation wells for reliable parameter estimates. Long term drawdown data resulted in more reliable estimates than the combinations of drawdown and buildup data. Measurement errors were found to reduce the accuracy and reliability of the estimates. The algorithm employed to carry out the parameter estimation is shown in Appendix B.

RESULTS

The data of Table 1 were assumed to be the true data of a geopressured-geothermal reservoir whose pressures were to be matched. The data shown were considered to be at the reservoir's initial state. The match period in all runs was 200 days; pseudo-steady state (volumetric) depletion of the hypothetical reservoir would have been attained within this time. Several runs were made using various combinations of drawdown and buildup data. The combinations of drawdown-buildup data used include 200 days drawdown, 135 days of drawdown with 65 days of buildup (135/65) and 100 days of drawdown with 100 days of buildup (100/100). Estimations were made with several levels of measurement error included with the simulated performance data. The inclusion of the measurement error made the performance data more realistic since measurement errors are inherent in such data. The confidence limits of the point estimates and the simulated and predicted performance data were all obtained at 95 percent confidence level. A 100 $(1-\alpha)$ percent confidence level implies that the probability of the true value being contained in the confidence interval is $(1-\alpha)$. The results of single-parameter estimations and multi-parameter estimations are presented in the following subsections.

SINGLE-PARAMETER ESTIMATION

The single parameter estimations were based on the assumption that all the reservoir parameters were known except for one which is to be estimated. In all the single-parameter estimations, the reservoir was considered to be homogeneous and isotropic. The reservoir was also considered to be elastic, that is, the uniaxial compaction coefficients during loading (drawdown) and unloading (buildup) were the same. The results are as follows:

a. INITIAL PERMEABILITY ESTIMATION

The results of permeability estimation for various levels of measurement error using all drawdown history data are shown in Table 2. The resolution used in the direct search segment of the estimation was 0.1; this implies 6 simulator runs (experiments) to obtain the optimum solution. It took a total of 13 experiments in each case to obtain the mean value (final estimate). This implies that 4 gradient iterations were required to bracket the optimum solutions (see Appendix B). The results were good in all cases as the point estimates were very close to the true values. The confidence intervals were narrow, less than one percent of the permeability true value, providing an evidence that the estimates were quite reliable. The reductions in criterion function from initial to final values were also impressive. It can be seen from Table 2 that a higher measurement error resulted in a less accurate point estimate and wider confidence interval, implying a less reliable point estimate.

For a higher measurement error, a wider confidence interval should be expected since the higher error would result in a higher parameter variance. Figure 1 shows pressures versus time at each gradient iteration during the history matching process. The values of permeability at the different gradient iterations are shown along with the corresponding criterion function values. The direct search segment of the estimation run was denoted as the fifth gradient iteration.

The results of permeability estimation using 135/65 and 100/100 drawdown-buildup data are presented in Tables 3 and 4 and in Figures 2 and 3 respectively. The resolution used was 0.1. The total number of simulator runs required for solution convergence was 13 in all cases of 135/65 data as in the cases of the drawdown data. However, two more experiments were required for the 100/100 data to converge. Drawdown-buildup data produced estimates that were less accurate and less reliable than those produced by the drawdown data. The estimates are still quite good, however. Although the point estimates were more accuarate for the 100/100 data for the 135/65 data, the confidence intervals were narrower in the latter case, meaning higher reliability. The reason for the lower drawdown-buildup ratio data yielding more accuarate point estimates may be due to the fact that the longer the well is shut-in, the closer the reservoir pressure approaches the initial pressure.

Shown in Figures 4 and 5 are the plots of the confidence limits of the history and predicted pressures, respectively, resulting from the single - parameter estimation of initial formation permeability. For the history period, the confidence interval is very narrow with the confidence limits of observed, true and computed pressures almost indistiguishable. The interval is also narrow for the predicted pressures, about 200 psi. The true pressures were contained in the confidence intervals.

b. UNIAXIAL COMPACTION COEFFICIENT ESTIMATION

The results of uniaxial compaction coefficient estimations using drawdown data are shown in table 5 and in Figure 6. The true value for the compaction coefficient is that of the hypothetical reservoir - 9.50 microsips. The initial estimate was 6.50 microsips in all cases. In the cases considered, 13 simulation runs were required just as in the case of permeability estimation runs. The compaction coefficient point estimates were close to the true value in all cases as shown in Table 5. The reason why the point estimates were these good is that the uniaxial compaction coefficient was assumed constant, that is, pressure independent. The accuracy of the point estimates decreased with an increase in the measurement error. The confidence intervals were narrow, implying reliable point estimates. The confidence intervals width increased with an increase in the measurement error.

Good estimates were obtained for the compaction coefficient when drawdown and buildup data were used. However, the results were not as good as those obtained when only drawdown data were used. The results indicated that the more drawdown data available for use, the more reliable the compaction coefficient estimates would be. Figures 7 and 8 show the confidence limits of the historical and predicted performance data, respectively. The intervals are seen to be narrow and the true pressures are contained in them in both cases.

c. INITIAL POROSITY ESTIMATION:

As shown in Table 6, nine simulator runs were required to estimate the initial formation porosity in all measurement error cases considered. The results in Table 6 and the pressure plot presented in Figure 9 were obtained using 200 days drawdown data. The resolution in these cases was 0.1 as in the other cases discussed above. The initial estimate used was 0.15 and the true value is 0.20. The point estimates obtained were very good as seen in the Table ranging from 0.199 for error-free data to 0.185 for 0.30 percent error data. The accuracy of the point estimates and the measurement error also have an inverse relationship in the porosity estimations. The effect of measurement error on the porosity point estimates is much more pronounced than the effect on permeability or compaction coefficient estimates. Notice how close the initial and final criterion function values are. There were no significant differences in the accuracy of the point estimates when combinations of drawdown and buildup data were matched.

Reliability studies indicated that while the reliability of the point estimates decreased with an increase in the measeurement error, the reliability was not affected by the type of data. Compared with those of permeability and compaction coefficient estimates, the confidence intervals for porosity point estimates were much wider, ranging up to 100 percent of the true porosity. The wide intervals is due to the mild sensitivity of pressure to porosity, resulting in low sensitivity coefficient and high porosity variance. As shown in Figures 10 and 11, the reliability of the history and predicted pressures in the porosity estimation case was high. The confidence intervals are still about 200 psi and the true pressures were contained in the intervals.

MULTI-PARAMETER ESTIMATIONS

In multi-parameter estimation runs, at least two of the parameters of interest - initial formation permeability, porosity and uniaxial compaction coefficient - were simultaneously estimated. Each run assumed that all the reservoir parameters except the combination to be jointly estimated were known. Some selected results are presented below.

a. INITIAL PERMEABILITY AND COMPACTION COEFFICIENT ESTIMATION:

The results for joint estimation of permeability and uniaxial compaction coefficient when 200 days drawdown and error-free data were used are presented in Table 7 and Figure 12. Seventeen simulator runs were required to obtain the optimal solution. The permeability estimate obtained was as good as that obtained by single-permeability estimation. However, the compaction coefficient point estimate obtained was not as good as the single-compaction coefficient estimate. With a true value of 9.50 microsips, the final estimate was 10.03 microsips. The point estimates obtained by the use of drawdown-buildup data were not as close to the true values as those obtained using 200 days drawdown data. Long production periods increased the reliability of the point estimates. The effect of measurement error was reduction in the accuracy of the estimates.

The individual confidence limits of the parameters at 95 percent confidence level are also shown in Table 7. Comparing these with the confidence limits of permeability and compaction coefficient single estimates, one finds that althought the true values are contained in the intervals of uncertainty, the intervals are wider in the former case implying reduced reliability. Figure 13 shows the joint confidence regions of permeability and uniaxial compaction coefficient estimates using 200 days drawdown data with 0.20 percent measurement error. Ellipses representing 75, 90, 95 and 99 percent confidence levels are concentric and are centered at A, the mean (final) estimates of the parameters. Point $\hat{A}$ represents the parameters' true values which are contained within 90, 95 and 99 percent confidence level regions. From Figure 13, one can see the extent to which the combination of the individual confidence limits deviated from the joint confidence region at the same confidence level. Notice that the individual 95 percent confidence level limits are represented by the rectangle PQRS. This has a greater area that the joint confidence region at the same confidence level. The orientation and shape of the ellipses of Figure 13 indicated that there is a strong interdependence between permeability and compaction coefficient and that compaction coefficient is less well determined. The confidence limits of the historical and some predicted pressures in the multi-parameter estimations of permeability and compaction coefficient and other multi-parameter estimations are shown in the work of Elemo (1978).

b. INITIAL PERMEABILITY AND POROSITY ESTIMATION:

Presented in Table 8 and Figure 14 are the results of the permeability and porosity joint estimation. As can be seen, the results are poor. After fifteen simulator runs, the mean values of permeability and porosity were 22.83 md and 0.188 respectively. These are about 14.14 and 11.25 percent deviations, for permeability and porosity respectively, of the final point estimates from their respective true values. The individual confidence limits of the permeability and porosity estimates are also shown in Table 8. The confidence intervals are wide, and porosity has an unrealistic interval. The joint confidence regions for this estimation is shown in Figure 15. Notice that the permeability estimate is more reliable than the porosity estimate because the true value is much farther away from the mean estimates along the porosity axis than the permeability axis. The orientation and shape of the ellipses indicated that there is a strong correlation between permeability and porosity and that porosity is less well determined than permeability. Although the estimate obtained for permeability in the permeability-porosity joint estimation is not as good as that obtained in the single-parameter estimation of permeability, the permeability estimate in the joint estimation is better than any of those obtained by using conventional well test methods (Elemo and Knapp,1979).

c. INITIAL COMPACTION COEFFICIENT AND POROSITY ESTIMATION:

Twenty-one simulator runs were made for closure of the compaction coefficient and porosity joint estimation. The compaction coefficient final estimate was fair, but that of porosity was poor. The point estimates deteriorated with longer buildup and shorter drawdown data and with an increase in measurement errors. The results are shown in Table 9 and Figure 16 for 200 days drawdown, error-free data case. The individual confidence limits of the point estimates are also presented in Table 9. The ellipses obtained as joint confidence regions for the parameters, shown in Figure 17, indicated that porosity is less well determined than the compaction coefficient and that the degree of correlation between them is extreme.

d. INITIAL PERMEABILITY,COMPACTION COEFFICIENT AND POROSITY ESTIMATION:

The results for the three-parameter estimation when 200 days drawdown and error-free data were used are as shown in Table 10 and Figure 18. The permeability estimate was the most accurate and reliable while the porosity estimate was the least accurate and reliable. The level of correlation between these parameters is high. The accuracy of all the parameters were found to decrease significantly as less drawdown data were used. The confidence limits of the individual estimates are seen in Table 10 to be meaningless with negative lower limits and a porosity upper limit greater than one. The joint confidence regions, although not established because of the complexity in their interpretation, are expected to indicate that porosity is least well determined followed by compaction coefficient.

DISCUSSION

The results presented above indicated that the technique of history matching can be applied for parameter estimation in stress-sensitive reservoirs and that a reasonable degree of accuracy and reliability can be expected in most cases. The results should be carefully interpreted. In history matching with real data, the estimated parameter values are only "apparent" reservoir parameter values. There are numerous inherent sources of error and many unknown parameters in history matching such that determination of the actual parameter values may not be possible (Elemo, 1978). Parameters obtained by history matching are usually non-unique. There is no reported criterion function or optimization technique that guarantees uniqueness. However, the chance for unique parameters can be increased by preanalysing the performance data to be matched. It can be observed from the results that the multi-parameter estimations were more demanding computationally and that they yielded less accuarate and reliable results than the single-parameter estimations. These are mainly due to the high level of interdependence of the various parameters.

To obtain good results by history matching, temporal responses of the matched performance data must be sensitive to variations in the parameters to be estimated. Notice that the porosity point estimates from the multi-parameter estimations are poor and unreliable. These poor porosity estimates are due to the performance data's mild sensitivity to variations in porosity (Elemo and Knapp, 1981) since distinguishability closure criterion was used in the multi-parameter estimations as against resolution closure criterion used in the single-parameter estimations (see Appendix B). The importance of a priori information in history matching can not be overemphasized. With a priori information, better choices of upper and lower parameter constraints and initial parameter estimates can be made. A priori information is often difficult to obtain. When it is available, for example from well log interpretations, it should be used effectively.

In the estimations described above, perfectly elastic rock behavior was assumed. In a reservoir with elastic behavior, the sediment compressibilities are the same whether the well is opened for production (pore pressure decreasing) or it is closed-in for pressure buildup (pore pressure increasing). This assumption of perfect elasticity is not valid for stress-sensitive reservoirs such as those being studied (Gray and Thompson, 1978). Usually stress-sensitive reservoirs' formation porosity and permeability exhibit hysteresis behavior. That is, formation compaction in geopressured systems is irreversible.Several runs were made in this study which the drawdown and buildup compaction coefficients were assumed different. Good point estimates were obtained and can be seen in the work of Elemo (1978). The results of runs for heterogeneous reservoir parameter estimation have also been reported by Elemo (1978).

The method and algorithm reported in this paper were developed for stress-sensitive, compacting and geopressured-geothermal reservoir system. Nonetheless, the method and algorithm can be adapted for parameter estimation in normally pressured,stress-independent and non-compacting reservoir system. A look at Appendix B will convince the reader of this fact.

CONCLUSIONS AND RECOMMENDATIONS

The following general conclusions can be drawn from the results of this study.

1.- The technique of history matching can be used to estimate the initial parameters of geopressured-geothermal reservoirs with greater expectation of success than conventional well test mathods.

2.- Single-parameter estimations resulted in good matches and in accurate and reliable estimates of permeability,uniaxial compaction coefficient and porosity with drawdown or drawdown and buildup history performance data except when the measurement errors are high.

3.- Porosity estimates are most sensitive to measurement errors.

4.- In all cases considered, the initial permeability estimates were better than those obtained by use of the conventional well test methods (Elemo and Knapp, 1979).

5.- Permeability as low as 5 md and as high as 100 md can be well estimated by history matching. Lower permeability levels usually result in more accurate and reliable estimates. Therefore, the model should have general applicability.

6.- Drawdown data are generally preferred to drawdown-buildup data. However, the latter can also provide reasonable permeability and porosity estimates.

7.- There is a high degree of correlation between permeability, uniaxial compaction coefficient and porosity; hence, results from multi-parameter estimations were less reliable than those from single-parameter estimations. However, good estimates of permeability,drawdown and buildup uniaxial compaction coefficients can be obtained from multi-parameter estimation runs. Porosity estimates are generally poor when it is jointly estimated with any other parameter.

8.- The method and estimation algorithm reported in this paper,while developed for stress-sensitive, compacting and geopressured-geothermal reservoir system, can be adapted for parameter estimation in normally pressured,stress-independent and non-compacting reservoir system.

It is recommended that history matching technique should be given serious consideration when parameter estimation is to be made for reservoirs that are stress-sensitive such as undercompacted geopressured-geothermal reservoirs.

NOMENCLATURE

A	=	Drainage area, $miles^2$
C.L.	=	Confidence level, percent
C_m	=	Uniaxial compaction coefficient, psi^{-1} (10^6 microsips)
C_{rm}	=	Rock matrix compressibility, psi^{-1} (10^6 microsips)
E	=	Criterion function, psi-psi-days
ESTAR	=	Minimum E, psi-psi-days
h	=	Reservoir depth, ft.
K	=	Absolute permeability, md
K_o	=	Initial permeability, md
K_{rw}	=	Relative permeability to water
K'	=	Pseudo-permeability, $\frac{Lbm - md}{cp}$
KC	=	Gradient iteration counter (2 simulation runs)
m_i	=	Number of observation data from i^{th} well
n	=	Total number of wells in the reservoir
p	=	Pore pressure, psi
p^{obs}	=	Observed pressure data, psi
p^{cal}	=	Calculated pressure data, psi
p_o	=	Initial reservoir pressure, psi
q	=	Flow rate, STB/day
Q	=	Adjoint variable
S_w	=	Water saturation
t	=	Time, days
T	=	Reservoir temperature, degree F.
T_M	=	Match period, days

V_p = Block bulk volume, ft^3

W = Reservoir net pay thickness, ft

W_o = Initial reservoir net pay thickness, ft

ε = Performance data measurement error

ϕ = Porosity, fraction

ϕ_o = Initial porosity

ϕ' = Pseudo-porosity, Lbm

ρ_w = Water density, Lbm/ft^3

ρ_{sc} = ρ_w at standard conditions, $\frac{Lbm}{ft^3}$

μ_w = Water viscosity, cp

REFERENCES

[1] Y. Bard : *Nonlinear Parameter Estimation*, Academic Press, New York, New York (1974).

[2] C. Chavent, M. Dupuy and P. Lemmonnier : *History Matching by use of Optimal Control Theory*, Society of Petroleum Engineers Journal, (February, 1975), pp. 74-86, Trans., AIME, Vol. 259.

[3] K.H. Coats, J.R. Dempsey and J.H. Henderson : *A New Technique for Determining Reservoir Description from Field Performance Data*, Society of Petroleum Engineers Journal (March, 1970), pp. 66-74; Trans., AIME, Vol. 249.

[4] N.R. Draper and H. Smith : *Applied Regression Analysis*, John Wiley and Sons, Inc., New York (1966).

[5] R.C. Earlougher, Jr. : *Advances in Well Test Analysis*, Society of Petroleum Engineers of AIME Monograph, Vol. 5 (1977).

[6] R.O. Elemo : *Automatic History Matching of Geopressured-Geothermal Reservoirs Performance Data by use of Optimal Control Theory*, Ph.D. Dissertation, The University of Texas at Austin (December 1978).

[7] R.O. Elemo and R.M. Knapp : *Estimation of Stress-Sensitive Reservoirs' Initial Parameters by use of the Conventional Well Test Methods*, Paper Presented at the Fourth United States Gulf Coast Geopressured-Geothermal Conference: Research and Development, The University of Texas at Austin, Texas, (October 29-31, 1979).

[8] R.O. Elemo and R.M. Knapp : *Numerically Controlled Experiments on Initial Parameters for Stress-Sensitive Reservoirs*, Revista Técnica (1981), Vol. 4, No. 1.

[9] J.F. Evers and E. Soeiinah : *Transient Test and Long-Range Performance Predictions in Stress-Sensitive Gas Reservoirs*, Journal of Petroleum Technology (August 1977), pp. 140-150.

[10] K.E. Gray and T.W. Thompson : *Compaction Measurement on Texas Gulf Coast Sandstone and Shales*, Monthly Letter Reports (Dec. 1977 - Present).

[11] R.M. Knapp, O.F.Isokrari, S.K. Garg and J.W. Pritichett : *An Analysis of Production from Geopressured Aquifers*, Paper SPE 6825 Presented at the SPE-AIME Fall Meeting, Denver, Colorado (Oct. 9-12, 1977).

[12] J.L. Lions : *Optimal Control of Systems Governed by Partial Differential Equations*, Translated into English by S.K. Mitter, Springer-Verlag, New York, N.Y. (1971).

[13] C.S. Matthews and D.G. Russell : *Pressure Buildup and Flow Test in Wells*, Society of Petroleum Engineers of AIME Monograph, Vol. 1 (1967).

[14] D.W. Peaceman : *Interpretation of Well-block Pressures in Numerical Reservoir Simulation*, Society of Petroleum Engineers Journal(June 1978) pp.183-194.

[15] R. Raghavan, J.D.T. Scorer and F.G. Miller : *An Investigation by Numerical Methods of the Effect of Pressure-Dependent Rock and Fluid Properties on Well Flow Tests*, Society of Petroleum Engineers Journal (June 1972) pp. 267-275; Trans., AIME.

[16] V.F. Samaniego, W.E. Brigham and F.G. Miller : *An Investigation of Transient Flow of Reservoir Considering Pressure-Dependent Rock and Fluid Properties*, Society of Petroleum Engineers Journal (April 1977), pp. 140-150.

ACKNOWLEDGEMENT

The financial support for this study, furnished by the United States Energy Research and Development Administration and the University of Texas at Austin Center for Energy Studies, is gratefully acknowledged by the authors.

APPENDICES

APPENDIX A : MATHEMATICAL MODELS

The history matching problems in this study were formulated as optimal control problems; therefore, two major models were developed and solved. The reservoir model is the "state" system. The state system consisted of the partial differential equations describing the flow of fluid in the reservoir and their initial and boundary conditions. The optimization model consisted of the "adjoint" system of equation, directly derived from the state system, and the criterion function.

In this study, a deformable, anisotropic and heterogeneous geopressured-geothermal aquifer model was used. Single-phase and slightly compressible fluid flow in two areal dimensions was simulated. From the three major concepts of mass conservation, Darcy's law and equation of state, the momentum transport equation for the water phase in geopressured-geothermal aquifer was obtained. The momentum transport equation, initial condition and boundary conditions are shown in equations A.1 through A.4.

$$\bar{\nabla} \left[\rho_w \frac{K_{rw}}{\mu_w} K (\bar{\nabla} P - \rho_w g \bar{\nabla} h)\right] + \frac{\rho_{w_{sc}} q_w}{V_p} =$$

$$\left\{ \phi S_w \left(\frac{\partial \rho_w}{\partial P}\right)_T + (\rho_w S_w)\left[C_m + C_{rm}(1-\phi)\right]\right\} \frac{\partial P}{\partial t} + (\phi \rho_w) \frac{\partial S_w}{\partial t} + \left[\phi S_w \left(\frac{\partial \rho_w}{\partial T}\right)_P\right] \frac{\partial T}{\partial t} \quad (A.1)$$

$$P(X,Z,t=0) = P_o(X,Z) \tag{A.2}$$

$$\frac{\partial P}{\partial X} + \rho_w g \frac{\partial h}{\partial X} = 0 \text{ at } X=0 \text{ and } X = LX \tag{A.3}$$

$$\frac{\partial P}{\partial Z} + \rho_w g \frac{\partial h}{\partial Z} = 0 \text{ at } Z = 0 \text{ and } Z = LZ \tag{A.4}$$

Equations describing the constitutive relationship between the parameters of geopressured-geothermal reservoirs have been reported by Knapp et al (1977). The modified form of these equations, shown as equations A.5, A.6 and A.7, were used in this study.

$$K(P) = K_o \left[1 + \frac{C_m + C_{rm}}{1 - \phi(P)} (P-P_o)\right] \tag{A.5}$$

$$\phi(P) = \phi_o + (C_m + C_{rm})(1 - \phi_o)(P - P_o) \tag{A.6}$$

$$W(P) = W_o[1 + C_m(P - P_o)] \tag{A.7}$$

The finite difference method was used to solve the above model. The computer calculated well block pressures were converted to bottom hole flowing pressures by using the concept of effective wellbore radius (Peaceman, 1978). Computed shut-in and/or the bottom hole flowing pressures were matched with the simulated history performance data.

The adjoint system of equations were derived from the state system of equations [equations A.1 through A.4]. The state and adjoint equations are the same except that an adjoint variable, Q, was substituted for the state variable, P, and the adjoint system was solved backward in time.

Assumptions of an isothermal process and complete water saturation in the reservoir were made to obtain the final adjoint equation, equation A.8. The boundary conditions are shown as equations A.9, A.10 and A.11 while the final condition is as shown in equation A.12.

$$\bar{\nabla}\,(k'\bar{\nabla}Q) - 2\sum_{i=1}^{N}\sum_{j=1}^{M_i}(p^{obs} - p^{cal})\delta(t - t_j) = -\frac{\partial}{\partial t}(\phi' Q) - \frac{\partial}{\partial t}(C'Q) \qquad (A.8)$$

$$\frac{\partial Q}{\partial \gamma}\,(r_i,t) = 0 \text{ for } r \text{ in } dW_i \qquad (A.9)$$

$$\frac{\partial Q}{\partial n}\,(r,t) = 0 \text{ for } r \text{ in } d\omega \qquad (A.10)$$

$$\sum_{i=1}^{N}\int_{o}^{T_M}\int_{dW_i} K'\,\frac{\partial Q}{\partial n}\,\delta P_w\; d\delta dt = 0 \qquad (A.11)$$

$$Q(r,T_M) = 0 \qquad (A.12)$$

where,

$$K' = V_p\,\frac{\rho_w}{\mu_w}\,KK_{rw}$$

$$\phi' = V_p\,C_w\phi;\; C_w = \frac{\partial\rho_w}{\partial p}$$

$$C' = V_p\rho_w\,[C_m + C_{rm}(1-\phi)]$$

γ = a point on a well boundary

dW_i = boundary of i^{th} well

ω = Spartial domain

$d\omega$ = reservoir boundary

Notice that the term $\frac{\partial Q}{\partial \gamma}(r_i,t)$ is the tangential derivative of the adjoint variable at the point γ of the boundary dW of each well in the reservoir and the term $\frac{\partial Q}{\partial n}$ (r,t) is the normal derivative of the adjoint variable at the point β of the boundary $d\omega$ of the reservoir.

During the derivation of the adjoint system of equations, a continuous form of criterion function was defined; this is shown in equation A.13. The criterion function was to be minimized.

$$E = \int_0^{T_m} \sum_{i=1}^{n} \sum_{j=1}^{m_i} [P^{obs}(r_i,t) - P^{cal}(r_i,t)]^2 \, \delta(t - t_j)dt \qquad (A.13)$$

where,

$\delta(\xi)$ is the Dirac delta function (Elemo, 1978)

The gradient of the criterion function with respect to the various parameters to be estimated were also obtained in the process of deriving the adjoint system of equations. These are shown as equations A.14, A.15 and (A.16).

$$\frac{\delta E}{\delta K'} = -\int_0^{T_M} (\bar{\nabla} Q - \bar{\nabla} P)dt \qquad (A.14)$$

$$\frac{\delta E}{\delta \phi'} = \int_0^{T_M} P \frac{\partial Q}{\partial t} dt \qquad (A.15)$$

$$\frac{\delta E}{\delta C'} = \int_0^{T_M} P \frac{\partial Q}{\partial t} dt \qquad (A.16)$$

The necessary conditions for the optimal solution which implies the best match between history and calculated performance data are that each of equation A.14, A.15 and A.16 be reduced to zero. The gradient based method of steepest descent was used in conjunction with the necessary conditions for optimality to solve the optimization model described above. By the method of steepest descent, the value of a parameter, P, at any gradient iteration, KC, is given by equation A. 17.

$$X_p^{KC+1} = X_p^{KC} - \lambda^{KC} \frac{\partial E}{\partial X_p^{KC}} \qquad (A.17)$$

where,

$$X_p^1 = X_{p_i} \quad \text{(initial value of the parameter)}$$

$$\lambda^{KC+1} = \nu\lambda^{KC}; \lambda^1 = \lambda_i$$

$$\frac{\partial E}{\partial X_p^{KC}} \quad \text{are from equations A.14 through A.16}$$

A more detailed description of the above models and their solutions are shown by Elemo (1978). The reliability study equations can also be found in the work.

APPENDIX B: PARAMETER ESTIMATION PROCEDURE

The following step by step algorithm was followed in the process of parameter estimation.

(a) Start with initial estimates, x_{p_i}, of the unknown parameter and the initial step size along the steepest descent directions desired, λ_i; set the number of gradient iteration counter to zero.

(b) Solve the state system of equations(equation A.1 through A.7) using the current values of the parameter estimates.

(c) Using the computed performance data from above and the history performance data, compute the value of the criterion function (equation(13)).

(d) Obtain the distinguishability, $\tau_1 = |E^{KC+1} - E^{KC}]$

(e) If the distinguishability is less than the set minimum or the intervals of uncertainty containing the optimum parameter values are obtained go to step (j); otherwise, proceed with step (f).

(f) If the number of gradient iterations exceed a set maximum, go to step (n).

(g) Solve the adjoint system of equations (equations A.8 through A.12).

(h) Using the solutions from steps (b) and (g), compute the gradients (equations A.14, A.15 and A.16).

(i) With the computed gradients, compute new and improved parameter values (equation A.17); advance gradient iteration counter by one and go to step (b).

(j) Reduce the intervals of uncertainty iteratively until the optimum parameter values are obtained.

(k) Compute the confidence limits of the parameter point estimates and the history and predicted performance data { Equations are presented by Elemo (1978) }.

(l) Print the results.

(m) STOP.

Step (c) of the above algorithm was performed by using the numerical integration technique of trapezoidal rule. Step (j) of the algorithm was performed by the Golden Section direct search method for the single-parameter estimations and the Hooke and Jeeves Pattern Search method for the multi-parameter estimations.

TABLE 1

BASE CASE DATA FOR A HYPOTHETICAL
GEOPRESSURED-GEOTHERMAL RESERVOIR

INITIAL PRESSURE	=	10,000 psi
INITIAL TEMPERATURE	=	300°F- ISOTHERMAL PROCESS
PRODUCTION RATE	=	40,000 STD/DAY
LENGTH	=	5,280 FEET
WIDTH	=	5,280 FEET
THICKNESS	=	100 FEET
WELLBORE RADIUS	=	3 INCHES
PERMEABILITY	=	20 MD
POROSITY	=	0.20
DENSITY	=	59.401 LBM/FT^3
DENSITY (STANDARD CONDITIONS)	=	62.757 LBM/FT^3
VISCOSITY	=	0.1993 CP
FORMATION VOLUME FACTOR	=	1.00 RES BBL/STB
UNIAXIAL COMPACTION COEFFICIENT	=	9.50×10^{-6} PSI^{-1}
ROCK MATRIX COMPRESSIBILITY	=	0.50×10^{-6} PSI^{-1}
FLUID COMPRESSIBILITY	=	3.04×10^{-6} PSI^{-1}

MEASUREMENT ERROR (%)	TOTAL NUMBER OF EXPERIMENTS	INITIAL VALUE OF E (PSI-PSI-DAYS)	FINAL VALUE OF E (PSI-PSI-DAYS)	LOWER CONFIDENCE LIMIT	MEAN VALUE	UPPER CONFIDENCE LIMIT
0.00	13	5.8288×10^{8}	2.2905×10^{2}	19.9932992	19.9962668	19.9992344
0.10	13	5.8208×10^{8}	1.9741×10^{4}	19.9643038	19.9918419	20.0193800
0.20	13	5.8133×10^{8}	7.8329×10^{4}	19.9325869	19.9874168	20.0422467
0.30	13	5.8061×10^{8}	1.7599×10^{5}	19.9008408	19.9829913	20.0651417

TABLE 2 · Confidence Limits of Permeability Estimates Using Different Measurement Errors

True Value = 20.0 md
Initial Value = 15.0 md
Confidence Level = 95.0 Percent
Type of Data · All Drawdown

MEASUREMENT ERROR (%)	TOTAL NUMBER OF EXPERIMENTS	INITIAL VALUE OF E (PSI-PSI-DAYS)	FINAL VALUE OF E (PSI-PSI-DAYS)	LOWER CONFIDENCE LIMIT	MEAN VALUE	UPPER CONFIDENCE LIMIT
0.00	13	4.9455×10^{8}	2.321×10^{4}	19.922138	19.9547311	19.987124
0.10	13	4.9381×10^{8}	4.1980×10^{4}	19.906776	19.9503189	19.993861
0.20	13	4.9311×10^{8}	9.9820×10^{4}	19.878798	19.9459062	20.013014
0.30	13	4.9245×10^{8}	1.9672×10^{4}	19.847327	19.9414926	20.035658

TABLE 3 : Confidence Limits of Permeability Estimates Using Different Measurement Errors

True Value = 20.0md
Initial Value = 15.0md
Confidence Level = 95.0 percent
Type of Data : 135 days Drawdown
65 days Buildup

MEASUREMENT ERROR (%)	TOTAL NUMBER OF EXPERIMENTS	INITIAL VALUE OF E (PSI-PSI-DAYS)	FINAL VALUE OF E (PSI-PSI-DAYS)	LOWER CONFIDENCE LIMIT	MEAN VALUE	UPPER CONFIDENCE LIMIT
0.00	15	4.3618×10^{8}	1.1051×10^{2}	20.001498	20.003885	20.006272
0.10	15	4.3540×10^{8}	1.9530×10^{4}	19.965324	19.9970412	20.028758
0.20	15	4.3466×10^{8}	7.7867×10^{4}	19.926910	19.990198	20.053485
0.30	15	4.3396×10^{8}	1.7512×10^{5}	19.888510	19.9833542	20.078199

TABLE 4 : Confidence Limits of Permeability Estimates Using Different Measurement Errors

True Value = 20.0md
Initial Value = 15.0md
Confidence Level = 95.0 percent
Type of Data : 100 days Drawdown
100 days Buildup

MEASUREMENT ERROR (%)	TOTAL NUMBER OF EXPERIMENTS	INITIAL VALUE OF E (PSI-PSI-DAYS)	FINAL VALUE OF E (PSI-PSI-DAYS)	LOWER CONFIDENCE LIMIT	MEAN VALUE	UPPER CONFIDENCE LIMIT
0.00	13	2.1515×10^{7}	1.6549×10^{2}	9.5050×10^{-6}	9.5035×10^{-6}	9.5220×10^{-6}
0.10	13	2.1425×10^{7}	1.9667×10^{4}	9.3989×10^{-6}	9.4915×10^{-6}	9.5841×10^{-6}
0.20	13	2.1376×10^{7}	7.8739×10^{5}	9.3001×10^{-6}	9.4851×10^{-6}	9.6700×10^{-6}
0.30	13	2.1365×10^{7}	1.7713×10^{5}	9.1954×10^{-6}	9.4720×10^{-6}	9.7485×10^{-6}

TABLE 5 : Confidence Limits of Uniaxial Compaction Coefficient Estimates Using Different Measurement Errors

True Value = $9.5 \times 10^{-6} psi^{-1}$
Initial Value = $6.5 \times 10^{-6} psi^{-1}$
Confidence Level = 95.0 percent
Type of Data All Drawdown

MEASUREMENT ERROR (%)	TOTAL NUMBER OF EXPERIMENTS	INITIAL VALUE OF E (PSI-PSI-DAYS)	FINAL VALUE OF E (PSI-PSI-DAYS)	LOWER CONFIDENCE LIMIT	MEAN VALUE	UPPER CONFIDENCE LIMIT
0.00	9	6.9414×10^{3}	9.0723	0.1975309	0.1989357	0.2003405
0.10	9	2.5416×10^{4}	1.9747×10^{4}	0.1296354	0.1937694	0.2579035
0.20	9	8.3534×10^{4}	7.9023×10^{4}	0.0654718	0.1937694	0.3220670
0.30	9	1.8130×10^{5}	1.7781×10^{5}	-0.0006514	0.1854102	0.3714718

Table 6 : Confidence Limits of Porosity Estimates Using Different Measurement Errors

True Value = 0.20
Initial Value = 0.15
Confidence Level = 95.0 percent
Type of Data : All Drawdown

PARAMETER	TRUE VALUE OF PARAMETER	INITIAL VALUE OF PARAMETER	LOWER CONFIDENCE LIMIT	MEAN VALUE	UPPER CONFIDENCE LIMIT
K(md)	20.0	15.0	19.779408	20.0012497	20.0223092
$C_m(psi^{-1})$	9.5×10^{-6}	6.5×10^{-6}	9.1864×10^{-6}	10.0254×10^{-6}	10.8645×10^{-6}

Table 7 : Individual Confidence Limits of Permeability and Uniaxial Compaction Coefficient Joint Estimates

Total No. of Experiments = 17
Initial Value of E = 7.8010×10^{8}
Final Value of E = 2.7954×10^{5}
Measurement Error = 0.0 percent
Confidence Level = 95.0 percent
Type of Data : All Drawdown

PARAMETER	TRUE VALUE OF PARAMETER	INITIAL VALUE OF PARAMETER	LOWER CONFIDENCE LIMIT	MEAN VALUE	UPPER CONFIDENCE LIMIT
K(md)	20.00	15.00	18.993759	22.827655	26.661550
ϕ	0.20	0.15	-5.3260158	0.1875000	5.7010158

Table 8 : Individual Confidence Limits of Permeability and Porosity Joint Estimates

Total No. of Experiments = 15
Initial Value of E = 5.8330×10^8
Final Value of E = 7.9329×10^7
Measurement Error = 0.0 percent
Confidence Level = 95.0 percent
Type of Data : All Drawdown

PARAMETER	TRUE VALUE OF PARAMETER	INITIAL VALUE OF PARAMETER	LOWER CONFIDENCE LIMIT	MEAN VALUE	UPPER CONFIDENCE LIMIT
$C_m(psi^{-1})$	9.50×10^{-6}	6.50×10^{-6}	9.0125×10^{-6}	9.8262×10^{-6}	10.6399×10^{-6}
ϕ	0.20	0.15	-0.3787668	0.140625	0.6600168

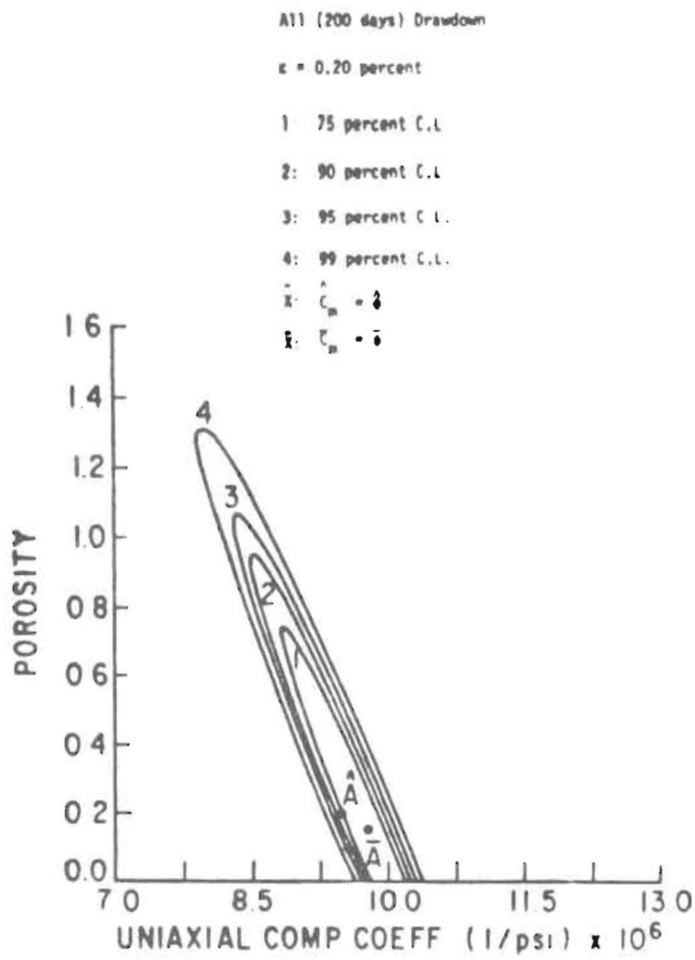
Table 9 : Individual Confidence Limits of Uniaxial Compaction Coefficient and Porosity Joint Estimates

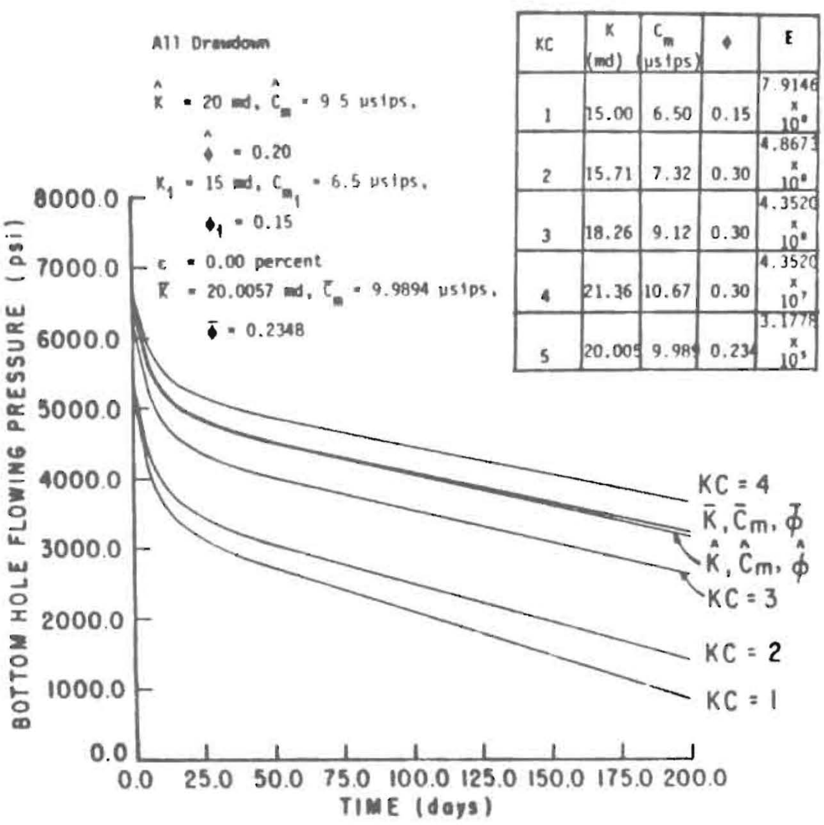
Total No. of Experiments = 21
Initial Value of E = 2.4071×10^7
Final Value of E = 6.3124×10^4
Measurement Error = 0.0 percent
Confidence Level = 95.0 percent
Type of Data : All Drawdown

PARAMETER	TRUE VALUE OF PARAMETER	INITIAL VALUE OF PARAMETER	LOWER CONFIDENCE LIMIT	MEAN VALUE	UPPER CONFIDENCE LIMIT
K(md)	20.0	15.00	15.484887	18.527208	21.569529
C_m (microsips)	9.50	6.50	-28.576899	8.469962	45.516823
ϕ	0.20	0.15	-16.509712	0.112500	16.734712

Table 10: Individual Confidence Limits of Permeability Compaction Coefficient and Porosity Joint Estimates

Total No. of Experiments = 19
Initial Value of E = 6.5436×10^{8}
Final Value of E = 4.0334×10^{7}
Measurement Error = 0.0 percent
Confidence Level = 95.0 percent
Type of Data : 135 Days Drawdown
65 Days Buildup


All (200 days) Drawdown
ε = 0.20 percent
1 75 percent C.L
2: 90 percent C.L
3: 95 percent C.L.
4: 99 percent C.L.
Â: Ĉm = φ̂
Ā: C̄m = φ̄
POROSITY
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
7.0
8.5
10.0
11.5
13.0
UNIAXIAL COMP COEFF (1/psi) x 10^6
4
3
2
1
Â
Ā


All Drawdown
K̂ = 20 md, Ĉm = 9.5 μsips,
φ̂ = 0.20
K1 = 15 md, Cm1 = 6.5 μsips,
φ1 = 0.15
ε = 0.00 percent
K̄ = 20.0057 md, C̄m = 9.9894 μsips,
φ̄ = 0.2348
KC | K (md) | Cm (μsips) | φ | E
1 | 15.00 | 6.50 | 0.15 | 7.9146 x 10
2 | 15.71 | 7.32 | 0.30 | 4.8673 x 10
3 | 18.26 | 9.12 | 0.30 | 4.3520 x 10
4 | 21.36 | 10.67 | 0.30 | 4.3520 x 10^7
5 | 20.005 | 9.989 | 0.234 | 3.1778 x 10^5
BOTTOM HOLE FLOWING PRESSURE (psi)
8000.0
7000.0
6000.0
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
0.0 25.0 50.0 75.0 100.0 125.0 150.0 175.0 200.0
TIME (days)
KC = 4
K̄, C̄m, φ̄
K̂, Ĉm, φ̂
KC = 3
KC = 2
KC = 1

All (200 days) Drawdown

ϵ = 0.20 percent

1: 75 percent C.L.

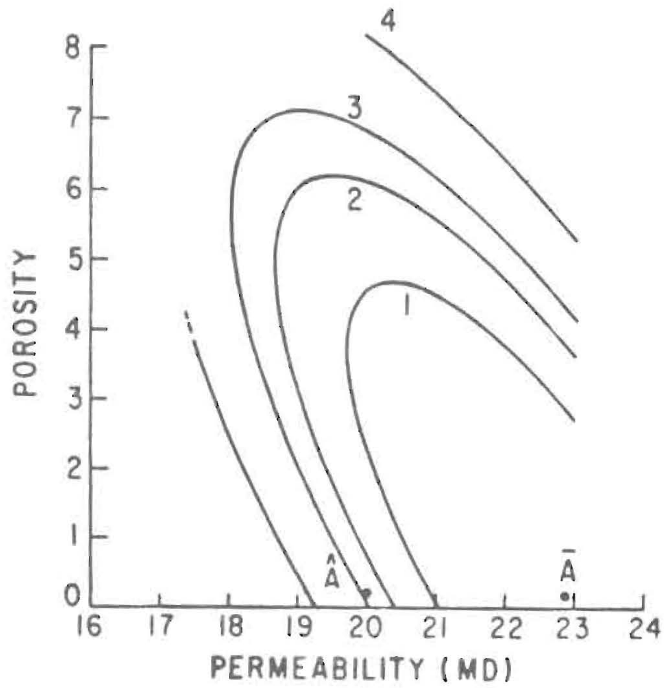
2: 90 percent C.L.

3: 95 percent C.L.

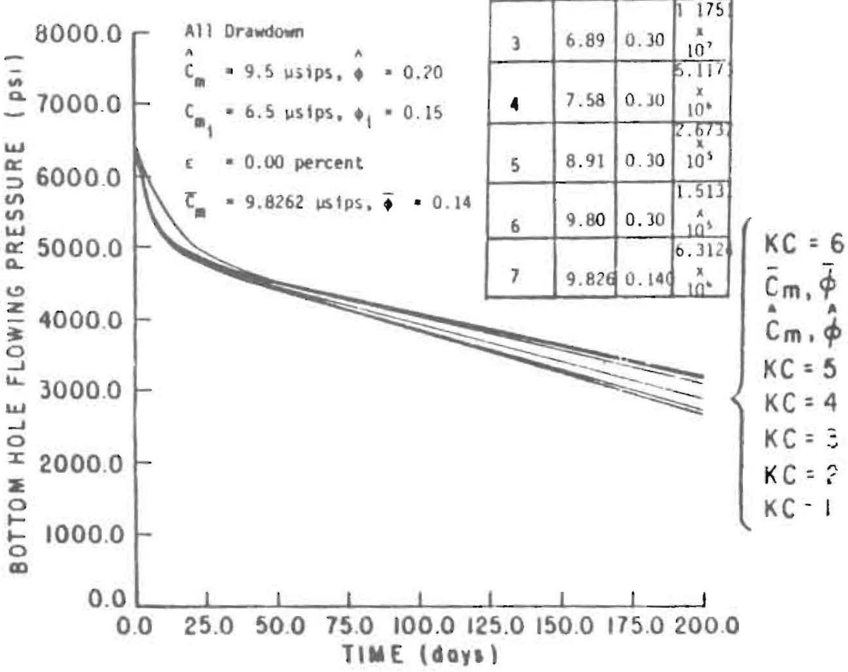
4: 99 percent C.L.

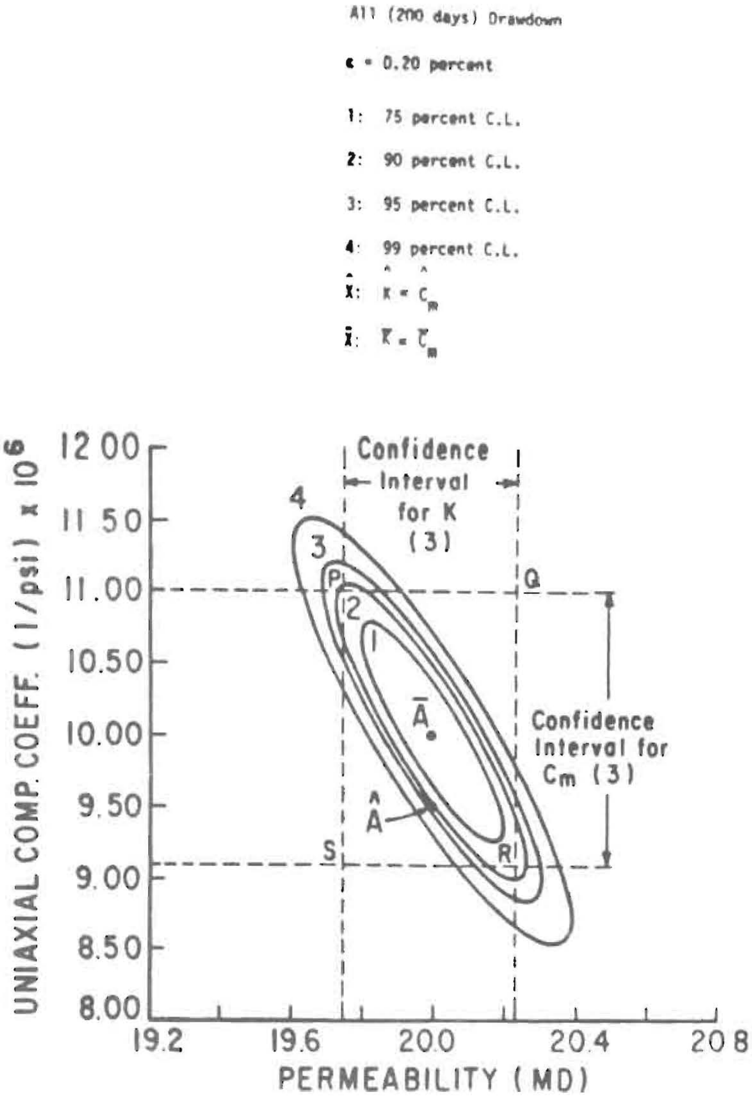
$\hat{x}$: $\hat{K} = \hat{\phi}$

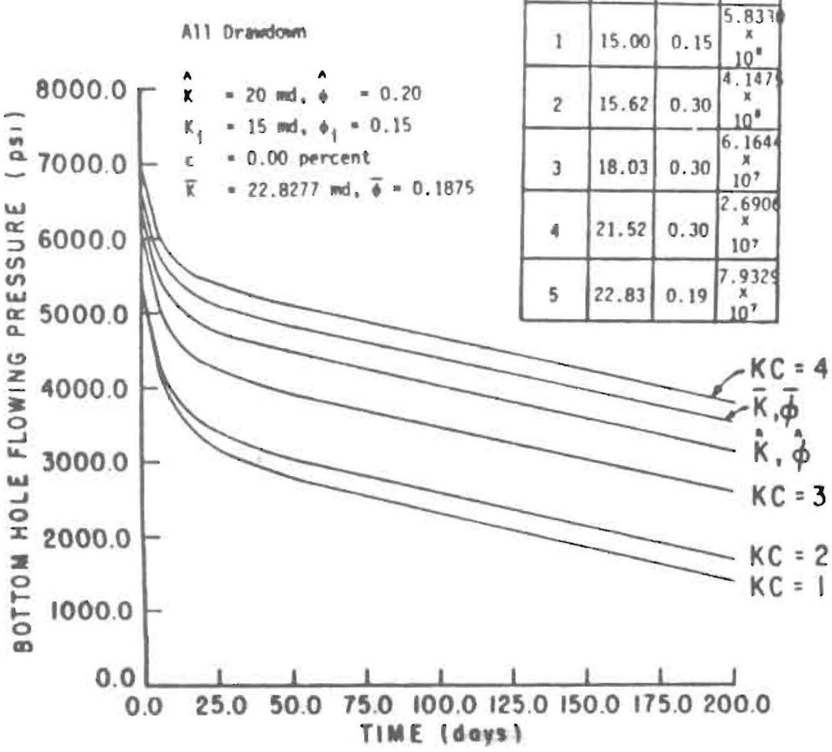
$\bar{x}$: $\bar{K} = \bar{\phi}$



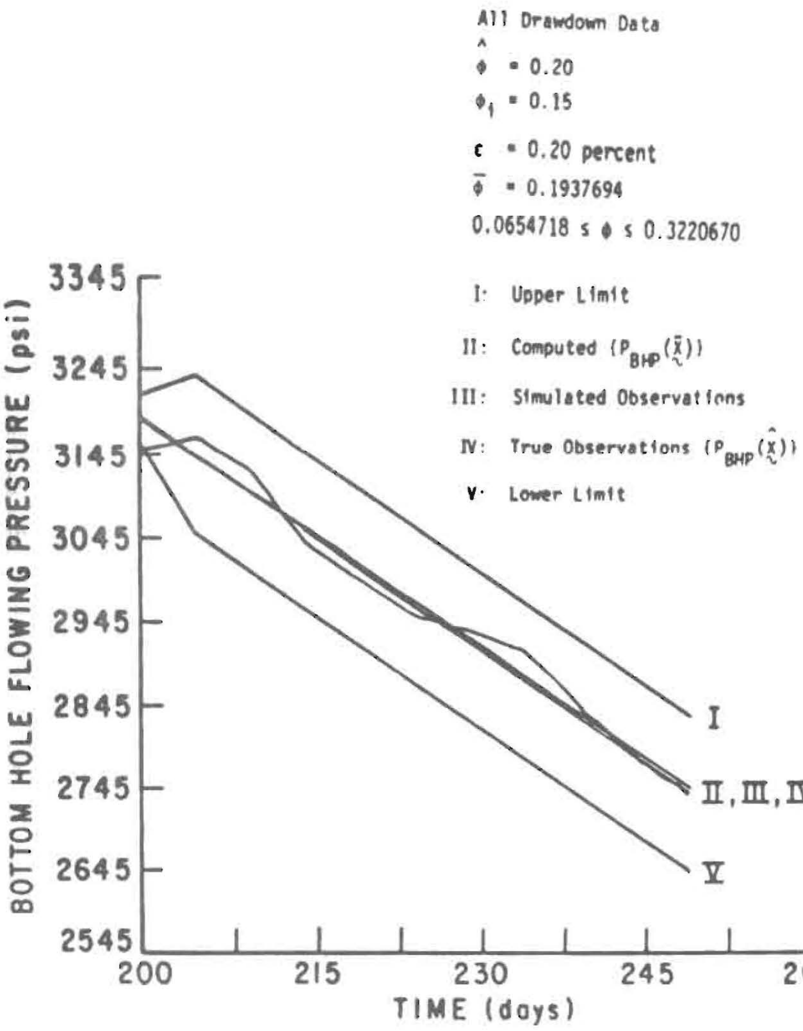
KC	C_m (μsips)	ϕ	E
1	6.50	0.15	2.4071 x 10^7
2	6.62	0.30	1.5541 x 10^7
3	6.89	0.30	1 1751 x 10^7
4	7.58	0.30	5.1173 x 10^6
5	8.91	0.30	2.6737 x 10^5
6	9.80	0.30	1.513 x 10^5
7	9.826	0.140	6.312 x 10^6

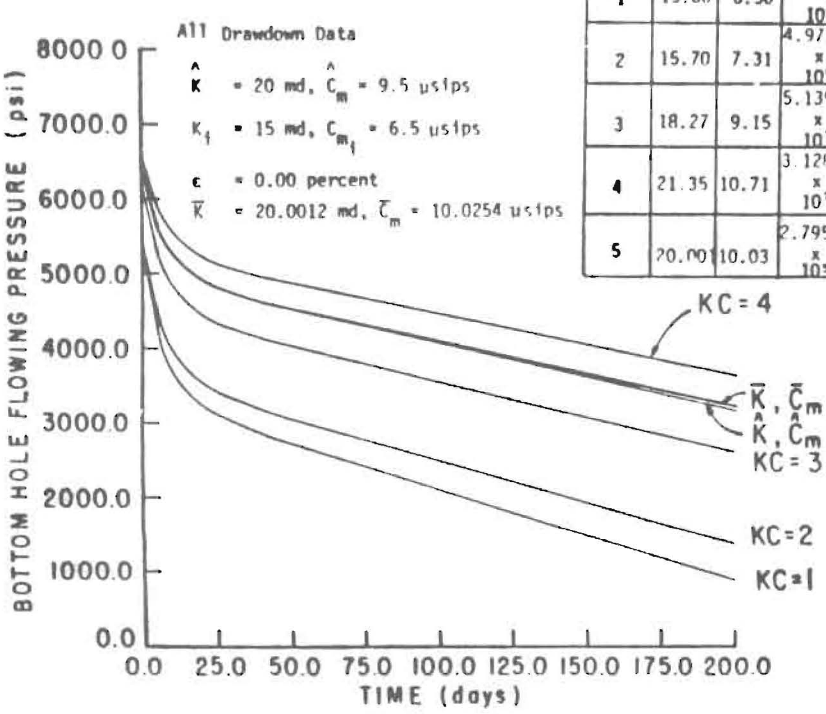




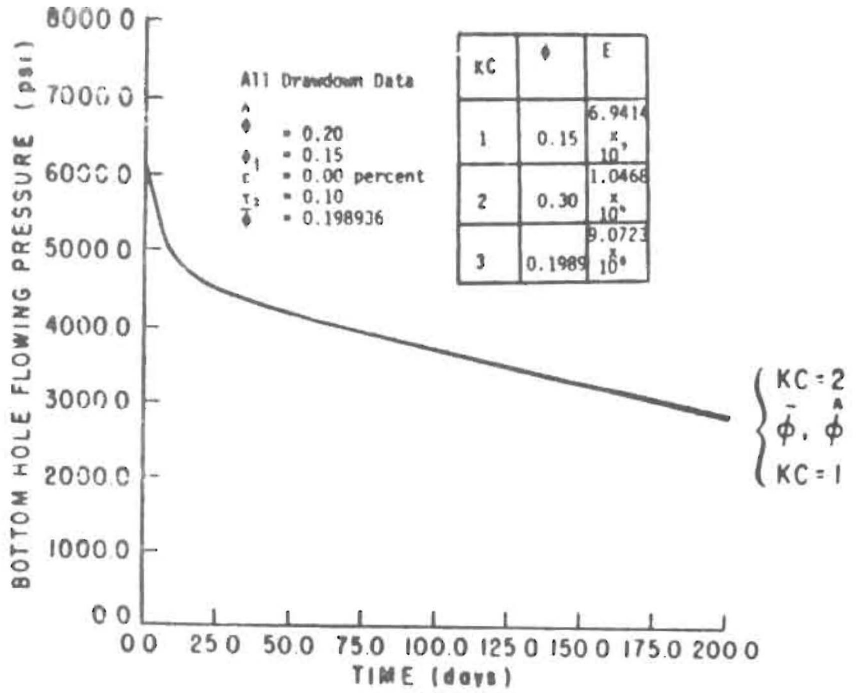


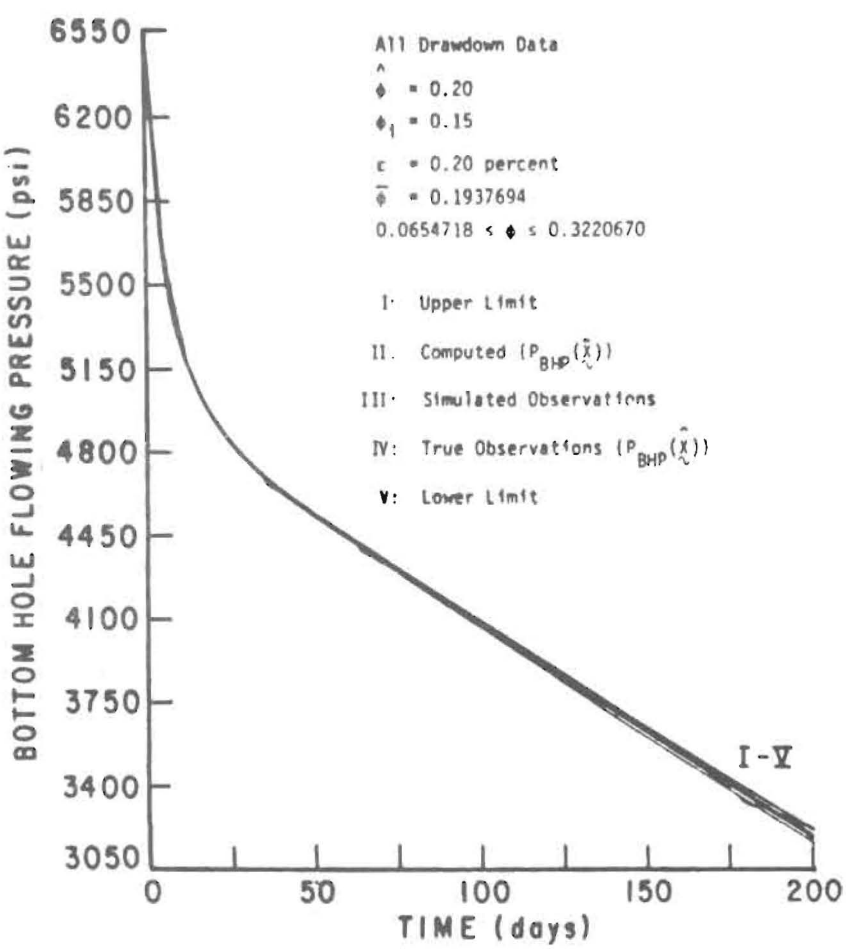
KC	K (md)	φ	E
1	15.00	0.15	5.833 x 10^8
2	15.62	0.30	4.147 x 10^8
3	18.03	0.30	6.164 x 10^7
4	21.52	0.30	2.690 x 10^7
5	22.83	0.19	7.932 x 10^7

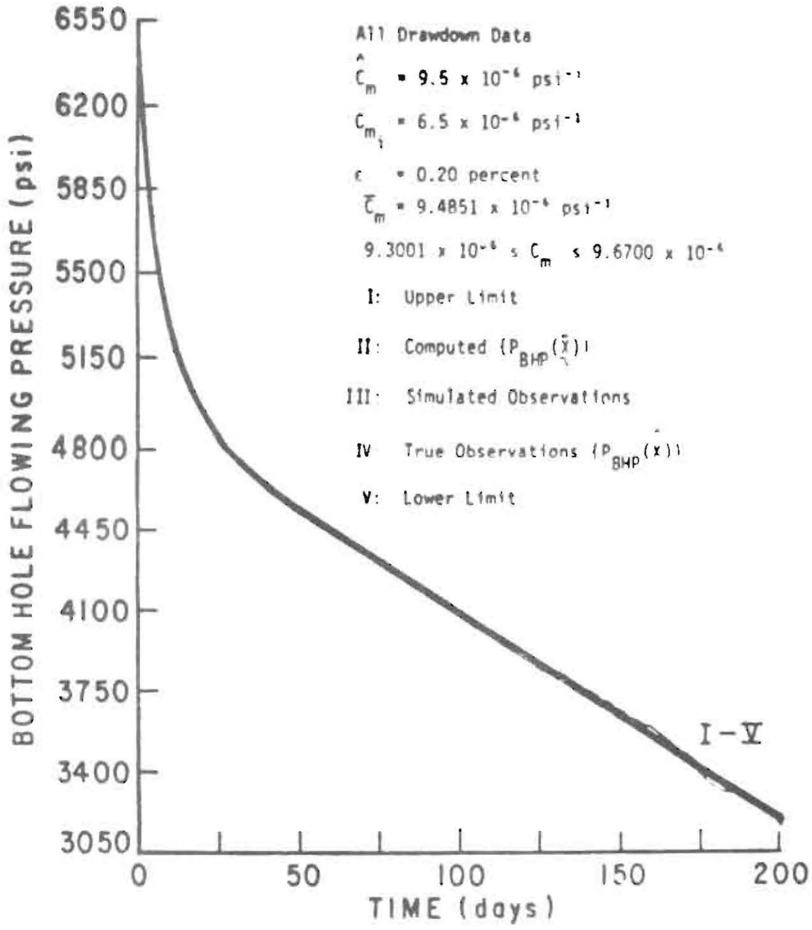


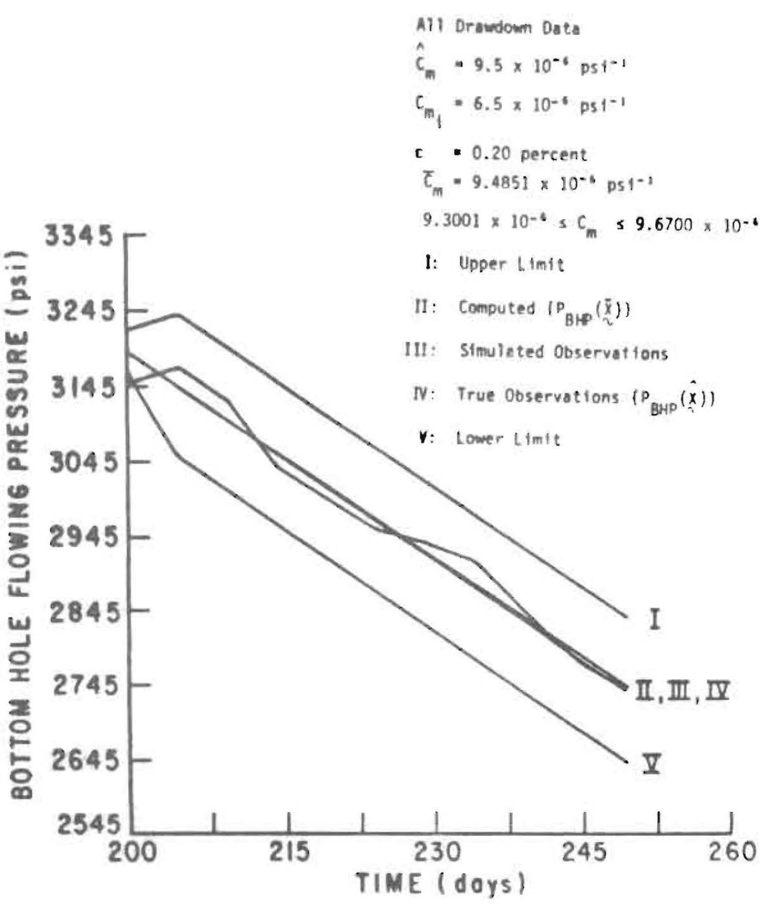


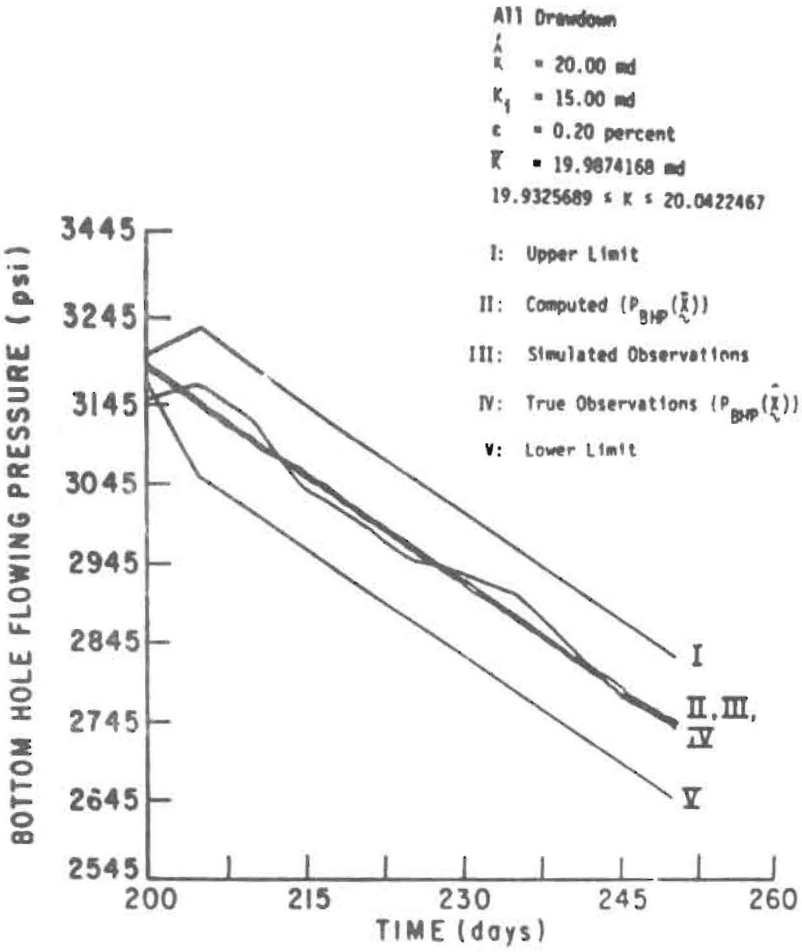
KC	K (md)	C_m μsips	E
1	15.00	6.50	7.8010 x 10^8
2	15.70	7.31	4.9735 x 10^8
3	18.27	9.15	5.1392 x 10^7
4	21.35	10.71	3.1284 x 10^7
5	20.001	10.03	2.7954 x 10^5

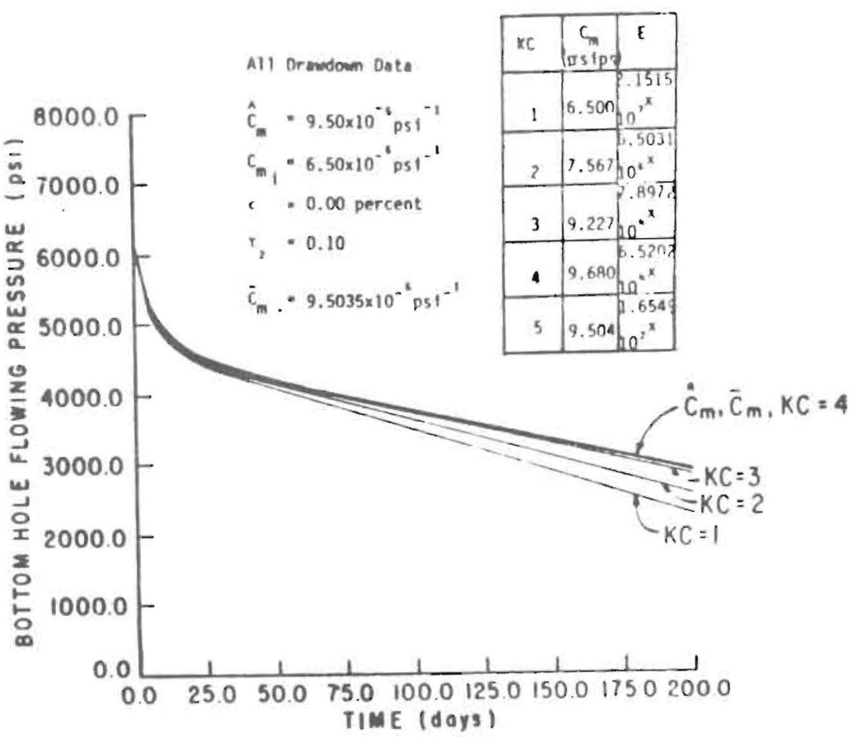

All Drawdown Data
$\hat{\phi}$ = 0.20
ϕ_1 = 0.15
ε = 0.00 percent
τ_2 = 0.10
$\bar{\phi}$ = 0.198936
KC
ϕ
E
1
0.15
6.9414 x 10
2
0.30
1.0468 x 10
3
0.1989
9.0723 x 10
KC = 2
$\bar{\phi}$, $\hat{\phi}$
KC = 1
BOTTOM HOLE FLOWING PRESSURE (psi)
TIME (days)

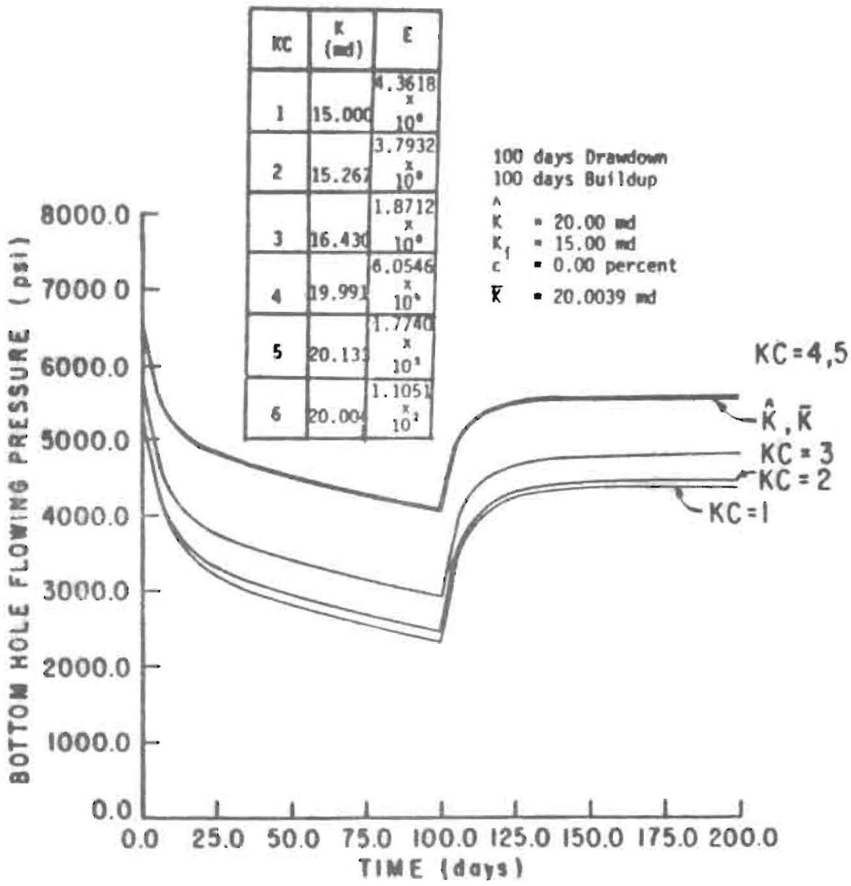

All Drawdown Data
$\hat{\phi}$ = 0.20
ϕ_1 = 0.15
ε = 0.20 percent
$\bar{\phi}$ = 0.1937694
0.0654718 ≤ ϕ ≤ 0.3220670
I: Upper Limit
II. Computed ($P_{BHP}(\tilde{X})$)
III: Simulated Observations
IV: True Observations ($P_{BHP}(\hat{X})$)
V: Lower Limit
I-V
BOTTOM HOLE FLOWING PRESSURE (psi)
TIME (days)

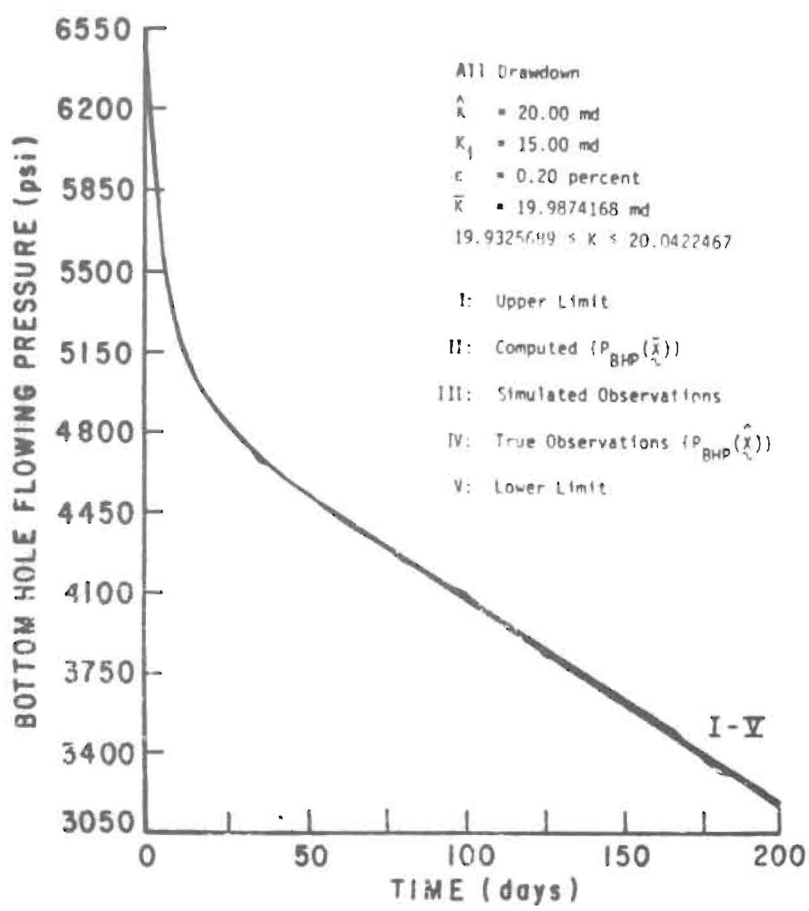

All Drawdown Data
Ĉm = 9.5 x 10⁻⁶ psi⁻¹
Cmi = 6.5 x 10⁻⁶ psi⁻¹
ε = 0.20 percent
C̄m = 9.4851 x 10⁻⁶ psi⁻¹
9.3001 x 10⁻⁶ ≤ Cm ≤ 9.6700 x 10⁻⁶
I: Upper Limit
II: Computed (PBHP(x̄))
III: Simulated Observations
IV: True Observations (PBHP(x̂))
V: Lower Limit
I-V
BOTTOM HOLE FLOWING PRESSURE (psi)
6550
6200
5850
5500
5150
4800
4450
4100
3750
3400
3050
0
50
100
150
200
TIME (days)

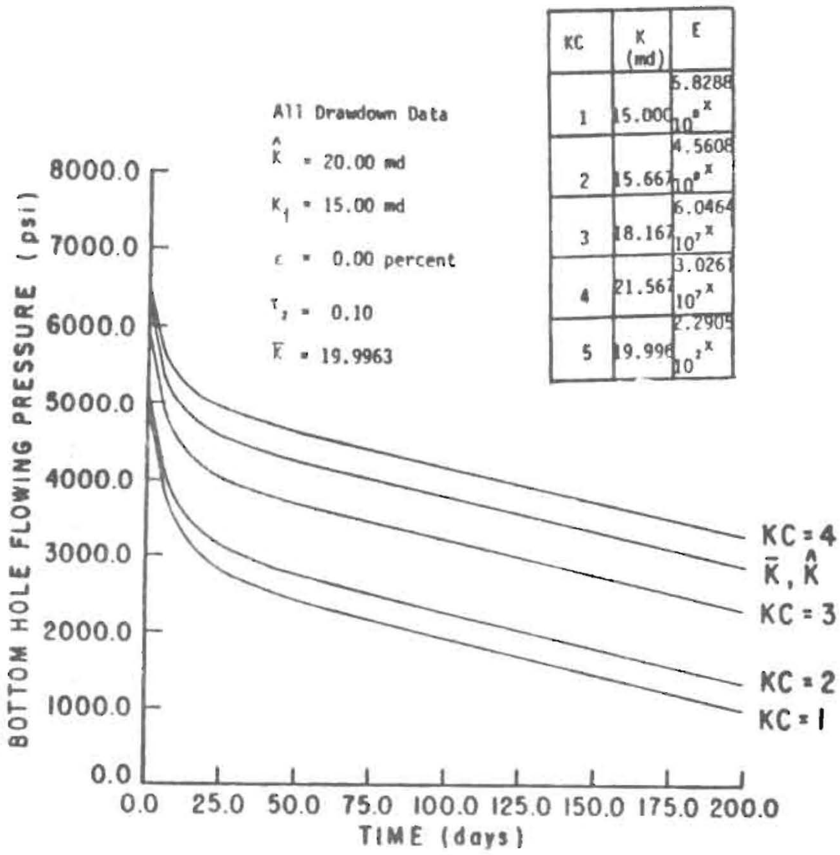

All Drawdown Data
Ĉm = 9.5 x 10⁻⁶ psi⁻¹
Cmi = 6.5 x 10⁻⁶ psi⁻¹
ε = 0.20 percent
C̄m = 9.4851 x 10⁻⁶ psi⁻¹
9.3001 x 10⁻⁶ ≤ Cm ≤ 9.6700 x 10⁻⁶
I: Upper Limit
II: Computed (PBHP(x̄))
III: Simulated Observations
IV: True Observations (PBHP(x̂))
V: Lower Limit
I
II, III, IV
V
BOTTOM HOLE FLOWING PRESSURE (psi)
3345
3245
3145
3045
2945
2845
2745
2645
2545
200
215
230
245
260
TIME (days)


All Drawdown
$\hat{K}$ = 20.00 md
K_i = 15.00 md
ε = 0.20 percent
$\bar{K}$ = 19.9874168 md
19.9325689 ≤ K ≤ 20.0422467
I: Upper Limit
II: Computed ($P_{BHP}(\bar{x})$)
III: Simulated Observations
IV: True Observations ($P_{BHP}(\hat{x})$)
V: Lower Limit
BOTTOM HOLE FLOWING PRESSURE (psi)
3445
3245
3145
3045
2945
2845
2745
2645
2545
200
215
230
245
260
TIME (days)
I
II, III, IV
V

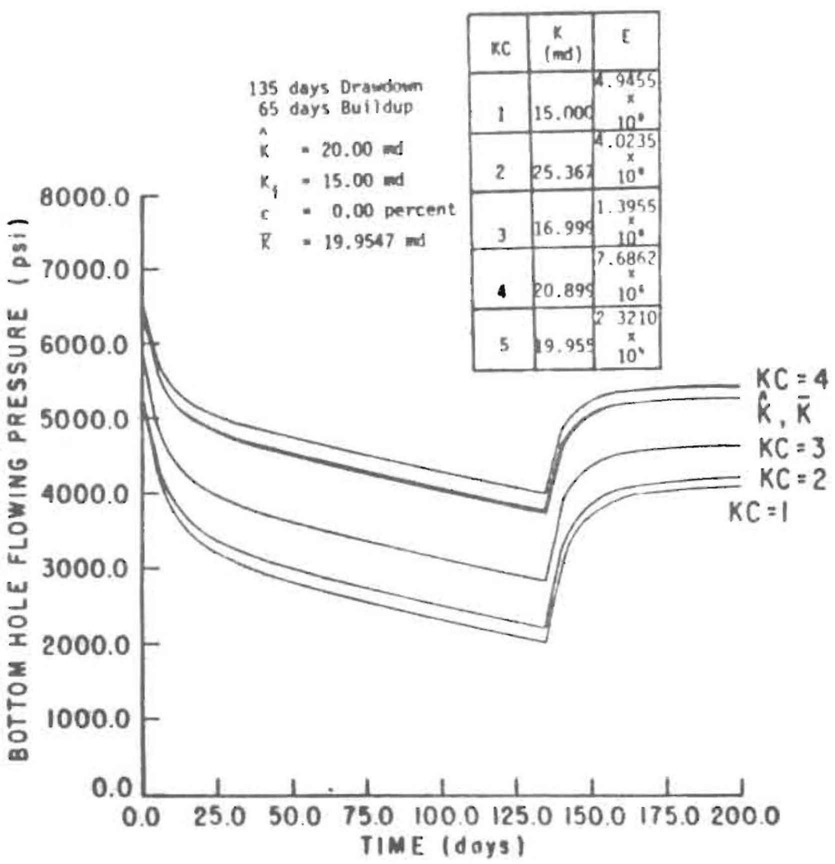

All Drawdown Data
$\hat{C}_m$ = 9.50x10^{-6} psi^{-1}
C_{m_i} = 6.50x10^{-6} psi^{-1}
ε = 0.00 percent
τ_2 = 0.10
$\bar{C}_m$ = 9.5035x10^{-6} psi^{-1}
KC | C_m (μsips) | E
1 | 6.500 | .1515 10^{7}x
2 | 7.567 | .5031 10^{6}x
3 | 9.227 | .8974 10^{4}x
4 | 9.680 | .5202 10^{4}x
5 | 9.504 | .6549 10^{2}x
BOTTOM HOLE FLOWING PRESSURE (psi)
8000.0
7000.0
6000.0
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
0.0
25.0
50.0
75.0
100.0
125.0
150.0
175.0
200.0
TIME (days)
$\hat{C}_m$, $\bar{C}_m$, KC = 4
KC=3
KC=2
KC=1


KC
K (md)
E
1 15.000 4.3618 x 10⁸
2 15.267 3.7932 x 10⁸
3 16.430 1.8712 x 10⁸
4 19.991 6.0546 x 10⁶
5 20.131 1.7740 x 10³
6 20.004 1.1051 x 10²
100 days Drawdown
100 days Buildup
$\hat{K}$ = 20.00 md
K_f = 15.00 md
c = 0.00 percent
$\bar{K}$ = 20.0039 md
KC=4,5
$\hat{K}$, $\bar{K}$
KC = 3
KC = 2
KC=1
BOTTOM HOLE FLOWING PRESSURE (psi)
8000.0
7000.0
6000.0
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
0.0 25.0 50.0 75.0 100.0 125.0 150.0 175.0 200.0
TIME (days)


All Drawdown
$\hat{K}$ = 20.00 md
K_f = 15.00 md
c = 0.20 percent
$\bar{K}$ = 19.9874168 md
19.9325689 ≤ K ≤ 20.0422467
I: Upper Limit
II: Computed ($P_{BHP}(\bar{x})$)
III: Simulated Observations
IV: True Observations ($P_{BHP}(\hat{x})$)
V: Lower Limit
I-V
BOTTOM HOLE FLOWING PRESSURE (psi)
6550
6200
5850
5500
5150
4800
4450
4100
3750
3400
3050
0 50 100 150 200
TIME (days)



KC	K (md)	E
1	15.000	5.8288 x 10^8
2	15.667	4.5608 x 10^8
3	18.167	6.0464 x 10^7
4	21.567	3.0261 x 10^7
5	19.996	2.2905 x 10^2



KC	K (md)	E
1	15.000	4.9455 x 10^8
2	25.367	4.0235 x 10^8
3	16.999	1.3955 x 10^8
4	20.899	7.6862 x 10^6
5	19.959	2.3210 x 10^5